The Consequences of An Unknown Debt Target

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MOTIVATION

- Agreement on benefits of central bank communication
- No consensus about conduct of fiscal policy
- Recently adopted fiscal rules:
  - EU Stability and Growth Pact sets debt target equal to 60%
  - Sweden 2010 Budget Act sets lending target of 1% of GDP
  - NZ Fiscal Responsibility Act requires “prudent” debt level
  - Canada committed to debt-to-GDP ratio of 25% by 2021
  - 1985 U.S. Gramm-Rudman-Hollings Balanced Budget Act
U.S. BUDGET PROPOSALS

CBO Alternative
CBO Pres Budget
Fiscal Commission
House Budget
CBO Baseline

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POLARIZATION OF THE U.S. CONGRESS

Polarization Index


0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

House Senate

Richter and Throckmorton: The Consequences of an Unknown Debt Target
Main Results

1. An unknown debt target amplifies the effects of tax shocks.
2. Stark changes in fiscal policy lead to welfare losses.
3. The Bush tax cut debate may have slowed the recovery.
RBC Model

Household chooses \( \{c_j, n_j, i_j, b_j\}_{j=t}^{\infty} \) to maximize

\[
E_t^\ell \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \log c_j - \chi \frac{n_j^{1+\eta}}{1 + \eta} \right\}
\]

subject to

\[
c_t + i_t + b_t = (1 - \tau_t)(w_t n_t + r_t^k k_{t-1}) + r_{t-1} b_{t-1} + \bar{z}
\]
\[
k_t = i_t + (1 - \delta) k_{t-1}
\]

P.C. firm produces \( y_t = \bar{a} k_{t-1}^{\alpha} n_t^{1-\alpha} \), and chooses \( \{k_{t-1}, n_t\} \) to maximize \( y_t - w_t n_t - r_t^k k_{t-1} \).
Fiscal Policy

- Government budget constraint,

\[ b_t + \tau_t(w_t n_t + r_t^k k_{t-1}) = r_{t-1} b_{t-1} + \bar{g} + \bar{z}. \]

- State-dependent income tax rate policy,

\[ \tau_t = \bar{\tau}(s_t) + \gamma(b_{t-1}/y_{t-1} - \bar{y}(s_t)) + \varepsilon_t, \]

where \( s \) is an \( m \)-state hidden Markov chain with transition matrix \( P \), and \( \varepsilon \sim \mathcal{N}(0, \sigma^2_\varepsilon) \).

- Signal extraction problem,

\[ x_t \equiv \tau_t - \gamma b_{t-1}/y_{t-1} = \bar{\tau}(s_t) - \gamma \bar{y}(s_t) + \varepsilon_t, \]

which has a mixed PDF of \( m \) normal distributions.
Sources of limited information

1. Time-varying mean, not standard deviation

2. Unknown debt target state
   - Bayesian updates conditional probabilities
   - Expectations formation is rational/Bayesian
   - Rational learning is embedded in optimization problem

3. Unknown transition matrix
   - Bayesian updates transition matrix
   - Expectations formation is adaptive
   - Household must reoptimize given estimate
# Information Sets

<table>
<thead>
<tr>
<th></th>
<th>Full Information Case 0</th>
<th>Limited Information Case 1</th>
<th>Limited Information Case 2</th>
</tr>
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<tbody>
<tr>
<td>Current Debt Target State</td>
<td>Known</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>Debt Target Transition Matrix</td>
<td>Known</td>
<td>Known</td>
<td>Unknown</td>
</tr>
</tbody>
</table>

\[
\mathbb{E} \left[ f(v_{t+1}, z_{t+1}^\ell, v_t, z_t^\ell) | \Omega_t^\ell \right] = 0
\]

\[
v_t \equiv (c_t, n_t, k_t, i_t, b_t)
\]

\[
z_t^\ell \equiv \begin{cases} (k_{t-1}, r_{t-1}b_{t-1}, \tau_t, s_t), & \text{for } \ell = 0, \\ (k_{t-1}, r_{t-1}b_{t-1}, \tau_t, q_{t-1}), & \text{for } \ell \in \{1, 2\}, \end{cases}
\]

\[
\Omega_0^t \equiv \{M, \Theta, z_0^t, P\}
\]

\[
\Omega_1^t \equiv \{M, \Theta, z_1^t, P\} \quad \Omega_2^t \equiv \{M, \Theta, z_2^t, \hat{P}_t, x_t\}
\]

\[
\Theta \equiv (\beta, \eta, \chi, \delta, \bar{a}, \alpha, \gamma, \{\bar{\tau}(i)\}_{i=1}^m, \{\bar{y}(i)\}_{i=1}^m, \sigma_\varepsilon^2)
\]

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\[ \mathbb{E} \left[ f(v_{t+1}, z_{t+1}^\ell, v_t, z_t^\ell) | \Omega_t^\ell \right] = \]

\[
\begin{cases}
\sum_{j=1}^{m} p_{ij} \int_{-\infty}^{+\infty} f(v_{t+1}, z_{t}^0, v_t, z_t^0) \phi(\varepsilon_{t+1}) d\varepsilon_{t+1} & \text{for } \ell = 0 \\
\sum_{i=1}^{m} q_t(i) \sum_{j=1}^{m} p^\ell_{ij} \int_{-\infty}^{+\infty} f(v_{t+1}, z_{t+1}^\ell, v_t, z_t^\ell) \phi(\varepsilon_{t+1}) d\varepsilon_{t+1} & \text{for } \ell \in \{1, 2\}
\end{cases}
\]

- For \( \ell = 0 \), \( s_t = i \) is known.
- For \( \ell \in \{1, 2\} \), \( q_t(i) \equiv \Pr[s_t = i | x^t] \).
- For \( \ell \in \{0, 1\} \), \( p_{ij} \in P \) is known.
- For \( \ell = 2 \), \( p_{ij} \in \hat{P}_t \) are estimates.
1. Recurring regime change: Aizenman and Marion (1993); Bizer and Judd (1989); Dotsey (1990)

2. Current regime unobserved:
   - Monetary: Andolfatto and Gomme (2003); Leeper and Zha (2003); Schorfheide (2005)

3. Other policy uncertainty: Davig and Leeper (2011); Davig et al. (2010, 2011); Richter (2012); Davig and Forester (2014); Bi et al. (2013)

4. Learning papers:
   - Adaptive: Kreps (1998); Cogley and Sargent (2008)
   - Bayesian: Schorfheide (2005); Bianchi and Melosi (2012)

5. Stochastic Volatility: Bloom (2009); Bloom et al. (2012)
   SV in Fiscal Policy: Fernández-Villaverde et al. (2013); Born and Pfeifer (2014)
## Calibration and Solution

<table>
<thead>
<tr>
<th></th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Debt Target</td>
<td>$by(1)$</td>
<td>0.60</td>
</tr>
<tr>
<td>Mid Debt Target</td>
<td>$by(2)$</td>
<td>0.75</td>
</tr>
<tr>
<td>High Debt Target</td>
<td>$by(3)$</td>
<td>0.90</td>
</tr>
<tr>
<td>Fiscal Policy Rule Coefficient</td>
<td>$\gamma$</td>
<td>0.30</td>
</tr>
<tr>
<td>Fiscal Noise Standard Deviation</td>
<td>$\sigma_\epsilon$</td>
<td>Estimated</td>
</tr>
<tr>
<td>Prior Transition Matrix</td>
<td>$\bar{P}$</td>
<td>Estimated</td>
</tr>
</tbody>
</table>

Debt targets are far apart so we use global nonlinear solution:

- Evenly spaced discretization
- Fixed-point policy function iteration
- Linear interpolation
- Gauss-Hermite integration
- 3-state Markov chain
American Taxpayer Relief Act: Top marginal tax rate increased 4.6 pp, payroll tax increased 2 pp
DATA AND TAX RULE FIT

Tax Rate

Debt-to-GDP ratio

Observations/Intercepts

Discretionary Tax Shock

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Estimation Results

- Estimating with Gibbs sampler gives $\hat{\sigma}_\varepsilon = 0.013$.
- The sampled average transition matrix and 68% credible interval are

\[
P_{16} = \begin{bmatrix}
0.78 & 0.11 & 0.05 \\
0.07 & 0.81 & 0.05 \\
0.07 & 0.12 & 0.66
\end{bmatrix}
\]

\[
\bar{P} = \begin{bmatrix}
0.81 & 0.12 & 0.07 \\
0.08 & 0.84 & 0.08 \\
0.10 & 0.18 & 0.72
\end{bmatrix}
\]

\[
P_{84} = \begin{bmatrix}
0.83 & 0.15 & 0.08 \\
0.10 & 0.87 & 0.11 \\
0.12 & 0.24 & 0.79
\end{bmatrix}
\]
1. Fiscal authority chooses $s_t$ and $\varepsilon_t$ to set $\tau_t$ given $b_{t-1}/y_{t-1}$

2. HH observes $x_t = \tau_t - \gamma b_{t-1}/y_{t-1}$ and in
   - Case 1 updates $q_{t-1}$ given $x_t$ with Bayes’ rule
   - Case 2 also updates $\hat{P}$ given $x^t$ with Gibbs sampler

3. In case 2, HH updates policy functions given $\hat{P}$

4. HH makes decisions conditional on information set, which updates $b_{t-1}/y_{t-1}$
SIMULATION PATHS

Case 1  Case 2

Output (%)  Consumption (%)

Capital (%)  Labor Hours (%)

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EFFECTS OF UNKNOWN STATE

Average Debt Target Inference versus Truth

Output (Case 1)

Discretionary Tax Shocks
DIFFERENCES IN OUTPUT

Output (Case 1)

Output (Case 2)

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Macroeconomic Uncertainty

- $y_t$ represents output in the model
- The expected value of the forecast error is given by
  \[ E_t[F E_{y,t+1}^\ell] = E_t[y_{t+1} - E_t y_{t+1} | \Omega_t^\ell] \]
- The expected volatility of the forecast error is
  \[ \sigma_{y,t}^\ell \equiv \sqrt{E_t[(F E_{y,t+1}^\ell - E_t[F E_{y,t+1}^\ell])^2 | \Omega_t^\ell]} \]
EXPECTED VOLATILITY OF OUTPUT

Case 1

Case 2

Output

SD of Output FE ($\sigma_y^\ell - \sigma_y^0$)

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FULL INFORMATION IRFs

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UNKNOWN STATE IRFs

Avg. Debt Target Inference

Tax Rate (% Point)

Debt/Output (% Change)

Output (% Change)

Consumption (% Change)

Real Interest Rate (% Point)

Labor Hours (% Change)

Capital (% Change)

Investment (% Change)
In Case 2, HH updates estimate of $P$ each period.

In period 1, their estimate is updated from

$$P = \hat{P}_0 = \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.90 & 0.05 \\ 0.05 & 0.05 & 0.90 \end{bmatrix}$$

to

$$\hat{P}_1 = \begin{bmatrix} 0.8947 & 0.0506 & 0.0547 \\ 0.0492 & 0.8970 & 0.0538 \\ 0.0454 & 0.0459 & 0.9087 \end{bmatrix}.$$
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Welfare Calculation

- Treat limited info. cases as alternative to full info.
- Solve for $\lambda^\ell$ that satisfies

$$E_t^\ell W(c_t(z^1_{t-1}), n_t(z^1_{t-1})) =$$

$$\begin{cases} 
\sum_{i=1}^m q_t(i)E_t^0 W((1 - \lambda^1)c_t(z^1_{t-1}|s_t = i), n_t(z^1_{t-1}|s_t = i)) & \text{Case 1} \\
\sum_{i=1}^m q_t(i) \sum_{j=1}^m \hat{p}_{ij} E_t^0 W((1 - \lambda^2)c_t(z^2_{t-1}|s_t, s_{t+1}), n_t(z^2_{t-1}|s_t, s_{t+1})) & \text{Case 2}
\end{cases}$$

- $\lambda^\ell > 0 \ (\lambda^\ell < 0)$ represents a welfare loss (gain) in case $\ell$
CASE 1 WELFARE DISTRIBUTION
(16-50-84 BANDS)
Welfare Gains and Losses

\[
\tau_t = \bar{\tau}(s_t) + \gamma \left( \frac{b_{t-1}}{y_{t-1}} - \bar{y}(s_t) \right) + \varepsilon_t,
\]

Scenario 1

Scenario 2
CASE 2 WELFARE DISTRIBUTION
(16-50-84 BANDS)
Assumption: People expected Bush tax cuts to sunset consistent with the goal of deficit reduction

Reality: Tax cuts were largely extended (projected to add $360B to annual deficit)

Suppose true debt target had always been high, despite Congress’ rally against debt

Hypothesis: People’s expectations were misaligned with the actual higher long-run debt target, which led to lower investment, output, and welfare loss
Debt Target is Hidden

(A) Contractionary paths

(B) Welfare costs

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Debt Target is Revealed

(A) Contractionary paths

(B) Welfare costs
CONCLUSION

1. An unknown debt target amplifies the effect of tax shocks through changes in expected tax rates
2. Unknown debt target leads to welfare losses on average
3. The Bush tax cut debate may have led to welfare losses
CBO Baseline Projections

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DISTRIBUTION OF DIFFERENCES
(25-50-75 QUANTILES)

Projection Year

Difference from Actual Debt-to-GDP (%)
Define a projection $g : \mathbb{R}^3 \to \mathbb{R}^2$, 

$$g(q_t) \equiv (q_t - o)B = \xi_t,$$

where $o$ is the origin and $\sum_i q_t(i) = 1$.

Apply the Gram-Schmidt process to obtain 

$$b_1 = \tilde{b}_1 = [0, 1, -1], \quad b_2 = \tilde{b}_2 - \text{proj}_{b_1} \left( \tilde{b}_2 \right) = [1, -1/2, -1/2],$$

so that $B \equiv [b_1^T/\|b_1\|, b_2^T/\|b_2\|]$ is an orthonormal basis.

The mapping becomes 

$$\xi_t(1) = q_t(2)(b_{21} - b_{11}) + q_t(3)(b_{31} - b_{11})$$

$$\xi_t(2) = q_t(2)(b_{22} - b_{12}) + q_t(3)(b_{32} - b_{12}).$$

where $b_{ij} \in B$. 

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HAMilton Filter

1. Calculate the joint probability of \((s_t = i, s_{t-1} = j)\),
\[
\Pr[s_t = i, s_{t-1} = j|x^{t-1}] = \Pr[s_t = i, s_{t-1} = j] \Pr[s_{t-1} = j|x^{t-1}].
\]

2. Calculate the joint conditional density-distribution,
\[
f(x_t, s_t = i, s_{t-1} = j|x^{t-1}) = f(x_t|s_t = i, s_{t-1} = j, x^{t-1}) \Pr[s_t = i, s_{t-1} = j|x^{t-1}].
\]

3. Calculate the likelihood of \(x_t\) conditional on its history,
\[
f(x_t|x^{t-1}) = \sum_{i=1}^{m} \sum_{j=1}^{m} f(x_t, s_t = i, s_{t-1} = j|x^{t-1}).
\]

4. Calculate the joint probabilities of \((s_t = j, s_{t-1} = i)\) conditional on \(x_t\),
\[
\Pr[s_t = i, s_{t-1} = j|x^t] = \frac{f(x_t, s_t = i, s_{t-1} = j|x^{t-1})}{f(x_t|x^{t-1})}.
\]

5. Calculate the output by summing the joint probabilities over the realizations \(s_{t-1}\),
\[
\Pr[s_t = i|x^t] = \sum_{j=1}^{m} \Pr[s_t = i, s_{t-1} = j|x^t].
\]
**IMPORTANCE SAMPLER**

- Posterior density is product of two independent Dirichlet distributions:

\[
f(P|s^T) \propto \left( \prod_{j=1}^{3} \Pi_j(P)^1j \right) \left( \prod_{i=1}^{3} \prod_{j=1}^{3} p_{ij}^{a_{ij}+m_{ij}^o-1} \right)
\]

where \( \pi \) is the stationary distribution of \( P \) and \( a \) are the initial shaping parameters.

- Sample \( L \) draws, \( \theta_{ij}^\ell \), from Dirichlet distribution, then weight them with \( w_\ell \equiv \prod_{j=1}^{3} \Pi_j(P_t^\ell)^1j \)

- \( \hat{p}_{ij} \) result from weighting procedure

\[
\hat{p}_{ij} = \frac{\sum_{\ell=1}^{L} w_\ell \theta_{ij}^\ell}{\sum_{\ell=1}^{L} w_\ell}.
\]
**Gibbs Sampler**

1. Initialize $s^T = \{s_1, \ldots, s_T\}$ by sampling from the prior, $P$.
2. For $t \in \{1, \ldots, T\}$ and $j \in \{1, 2, 3\}$, sample $s_t$
   - If $t = 1$, then $f(s_1|x^T, s_{-1}) \propto \Pi_j(P)p_{jk}f(x_1|s_1)$, where $s_2 = k$.
   - If $1 < t < T$, then $f(s_t|x^T, s_{-t}) \propto p_{ij}p_{jk}f(x_t|s_t)$, where $s_{t-1} = i$ and $s_{t+1} = k$.
   - If $t = T$, then $f(s_T|x^T, s_{-T}) \propto \Pi_j(P)p_{ij}f(x_T|s_T)$, where $s_{T-1} = i$.

   $\Pi_j(P)$ is the $j$th element of the stationary distribution of $P$, $f(x_t|s_t) = \exp \left\{ -\varepsilon_t^2/(2\sigma^2) \right\}/\sqrt{2\pi\sigma^2}$, where $\varepsilon_t = x_t - (\bar{\tau}(s_t) - \gamma by(s_t))$ is the discretionary i.i.d. tax shock.

3. Use the importance sampler to draw $P$ given $s^T$.
4. Repeat steps 2 and 3 $N$ times.