The Zero Lower Bound and Endogenous Uncertainty

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Introduction

- Considerable interest in understanding the relationship between uncertainty and economic activity
- Several recent papers find a negative relationship in the data using various measures of uncertainty
- DSGE literature uses stochastic volatility shocks to model uncertainty, which typically reduce economic activity
- Literature primarily focuses on how economic activity responds to changes in specific types of uncertainty
- This paper examines how recent events affected uncertainty and its correlation with real GDP growth

Finding in the Data

- Measures of uncertainty:
  - Time-varying VAR with stochastic volatility
  - Stock Market Volatility
  - Survey-based forecast dispersion
  - Macro uncertainty index from Jurado et al. (AER, 2015)
- A stronger negative correlation between real GDP growth and uncertainty emerged in the data in 2008Q4
- Theory: ZLB constraint contributed to the stronger negative correlation because it restricts the ability of the central bank to stabilize the economy

Testing our Theory

- Estimate a nonlinear DSGE model with a ZLB constraint
  - Small-scale new Keynesian model
  - Habit persistence, interest rate smoothing
  - Preference, growth, and monetary policy shocks
- Create a data-driven, forward-looking uncertainty measure
  - Expected volatility of real GDP growth forecast errors
- Results from our estimated model:
  - Correlations with GDP growth near the values in the data
  - Strong positive correlation between uncertainty measures
  - Cross correlations with leads/lags of GDP growth indicate uncertainty arises due to what is happening in the economy

Relationship between

Economic Activity and Uncertainty

VAR with Stochastic Volatility

- Following Primiceri (2005), we estimate a time-varying VAR with stochastic volatility using Bayesian methods:

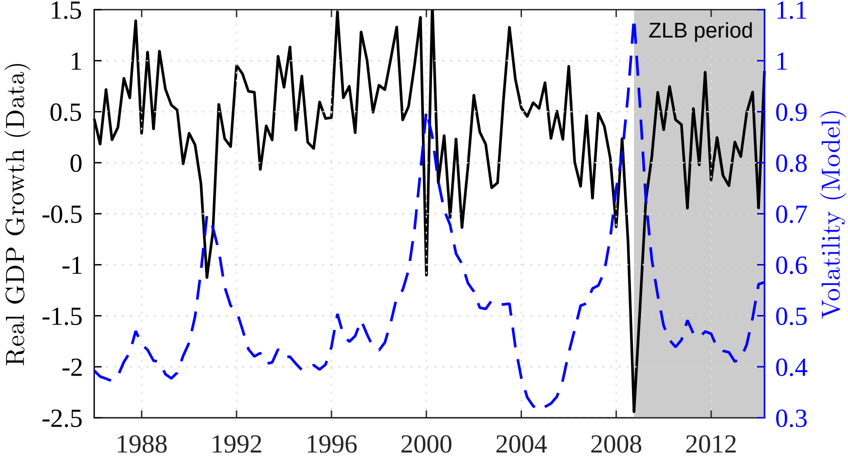
$$y_t = b_t + B_{1,t} y_{t-1} + B_{2,t} y_{t-2} + A_t^{-1} \Sigma_t \varepsilon_t, \quad t = 1, \ldots, T$$

where $y_t = \begin{bmatrix} \text{GDP Growth} & \text{Inflation} & \text{T-Bill} \end{bmatrix}'$, $\varepsilon \sim N(0,1)$,

$$A_t = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix}, \quad \text{and} \quad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{3,t} \end{bmatrix}.$$

- Calculate the correlation between real GDP growth and the standard deviation of the shock to output ($\sigma_{1,t}$) for each draw from the posterior distribution from 1986Q1-2014Q2

TVP VAR Estimated Volatility



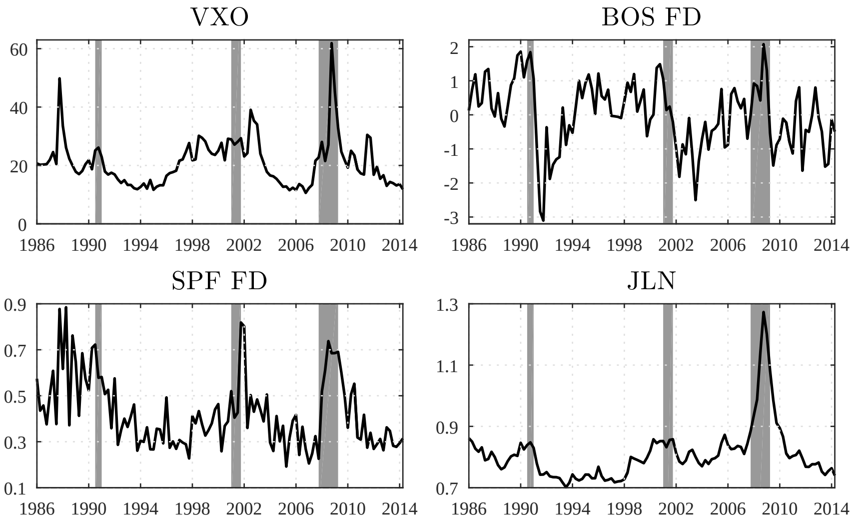
Real GDP vs. Estimated Volatility

	Pre-ZLB Sample (1986Q1-2008Q3)	ZLB Sample (2008Q4-2014Q2)	Differences (ZLB - Pre-ZLB)
Federal Funds Rate	−0.24**	−0.60***	−0.35*
Federal Funds Rate + Financial Uncertainty	−0.21*	−0.57**	−0.34*
Shadow Rate	−0.24**	−0.60***	−0.35*
10-Year T-Bill Rate	−0.25**	−0.61***	−0.35*
Real Rate	−0.23*	−0.59***	−0.36*

Alternative Measures of Uncertainty

- Real GDP is our main measure of economic activity
  - Also examine industrial production
- Popular measures of uncertainty:
  - CBOE VXO index
  - SPF real GDP forecast dispersion
  - BOS forecast dispersion
  - Jurado, Ludvigson, and Ng (JLN) Macro Uncertainty
- Quarterly data: 1986Q1-2014Q2
- For each measure, calculate correlations with GDP growth

Alternative Uncertainty Measures



RGDP Growth vs. Uncertainty

	VXO	BOS FD	SPF FD	JLN
Pre-ZLB Sample (1986Q1-2008Q3)	−0.09	−0.19**	−0.08	−0.45***
ZLB Sample (2008Q4-2014Q2)	−0.72***	−0.70***	−0.47**	−0.73***
Difference (ZLB−Pre-ZLB)	−0.64***	−0.51***	−0.39**	−0.29**

Fisher z-transformation—tests whether the pre- and post-Great Recession correlations are significantly different:

- 1% level: VXO and BOS FD
- 5% level: SPF FD and JLN

IP GROWTH VS. UNCERTAINTY

	VXO	BOS FD	SPF FD	JLN
		Quarterly Data		
Pre-ZLB Sample (1986Q1-2008Q3)	−0.11	−0.19**	−0.26***	−0.64***
ZLB Sample (2008Q4-2014Q2)	−0.74***	−0.59***	−0.61***	−0.78***
Difference (ZLB-Pre-ZLB)	−0.62***	−0.40**	−0.34**	−0.14
		Monthly Data		
Pre-ZLB Sample (1986Q1-2008Q3)	−0.10*	−0.13**	–	−0.41***
ZLB Sample (2008Q4-2014Q2)	−0.50***	−0.38***	–	−0.54***
Difference (ZLB-Pre-ZLB)	−0.40***	−0.25**	–	−0.13

Theoretical Model

New Keynesian Model

The representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log(c_t - hc_{t-1}^a) - \chi n_t^{1+\eta}/(1+\eta)],$$

where $\tilde{\beta}_0 \equiv 1$ and $\tilde{\beta}_t = \prod_{j=1}^{t} \beta_j$ for $t > 0$ subject to

$$c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t$$

Optimality implies

$$w_t = \chi n_t^{\eta}(c_t - hc_{t-1}^a),$$
$$1 = i_t E_t[q_{t,t+1}/\pi_{t+1}],$$

where $q_{t,t+1} \equiv \beta_{t+1}(c_t - hc_{t-1}^a)/(c_{t+1} - hc_t^a)$ is the pricing kernel.

New Keynesian Model

- Firm optimality condition:

$$\varphi\left(\frac{\pi_t}{\bar{\pi}}-1\right)\frac{\pi_t}{\bar{\pi}}=1-\theta+\theta\frac{w_t}{z_t}+\varphi E_t\left[q_{t,t+1}\left(\frac{\pi_{t+1}}{\bar{\pi}}-1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{y_t}\right]$$

- Production Function

$$y_t=z_tn_t$$

- Monetary policy rule

$$i_t=\max\{\underline{\imath},i_t^*\}$$
$$i_t^*=(i_{t-1}^*)^{\rho_i}(\bar{\imath}(\pi_t/\bar{\pi})^{\phi_\pi}(c_t/(\bar{g}c_{t-1}))^{\phi_c})^{1-\rho_i}\exp(\nu_t),$$

where i^* is the notional interest rate.

New Keynesian Model

- Resource constraint:

$$c_t = [1 - \varphi(\pi_t/\bar{\pi} - 1)^2/2]y_t = y_t^{gdp}$$

- Discount factor (β) follows an AR(1) process

$$\beta_t = \bar{\beta}(\beta_{t-1}/\bar{\beta})^{\rho_\beta} \exp(\varepsilon_t)$$

- Technology (z) follows a random walk:

$$z_t = z_{t-1} g_t$$
$$g_t = \bar{g}(g_{t-1}/\bar{g})^{\rho_g} \exp(\upsilon_t)$$

- Exogenous state variables: β_t, g_t, ν_t
- Endogenous state variables: c_{t-1}, i^*_{t-1}
- Policy functions: c_t, π_t

Competitive Equilibrium

Consists of sequences of quantities $\{\tilde{\lambda}_t, \tilde{c}_t, \tilde{y}_t\}_{t=0}^{\infty}$, prices $\{w_t, i_t^*, \pi_t\}_{t=0}^{\infty}$, and shocks $\{\beta_t, g_t\}_{t=0}^{\infty}$ that satisfy:

$$\tilde{\lambda}_t = \tilde{c}_t - h\tilde{c}_{t-1}/g_t$$

$$\tilde{w}_t = \chi \tilde{y}_t^{\eta} \tilde{\lambda}_t$$

$$1 = i_t E_t[\beta_{t+1}(\tilde{\lambda}_t/\tilde{\lambda}_{t+1})/(g_{t+1}\pi_{t+1})]$$

$$\varphi\left(\frac{\pi_t}{\bar{\pi}} - 1\right)\frac{\pi_t}{\bar{\pi}} = 1 - \theta + \theta\tilde{w}_t + \varphi E_t\left[\beta_{t+1}\frac{\tilde{\lambda}_t}{\tilde{\lambda}_{t+1}}\left(\frac{\pi_{t+1}}{\bar{\pi}} - 1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\right]$$

$$\tilde{c}_t = [1 - \varphi(\pi_t/\bar{\pi} - 1)^2/2]\tilde{y}_t$$

$$i_t^* = (i_{t-1}^*)^{\rho_i}(\bar{\imath}(\pi_t/\bar{\pi})^{\phi_\pi}(g_t\tilde{c}_t/(\bar{g}\tilde{c}_{t-1}))^{\phi_c})^{1-\rho_i}\exp(\sigma_\nu \nu_t)$$

$$g_t = \bar{g}(g_{t-1}/\bar{g})^{\rho_g}\exp(\varepsilon_t)$$

$$\beta_t = \bar{\beta}(\beta_{t-1}/\bar{\beta})^{\rho_\beta}\exp(\upsilon_t)$$

Solution Method

- Solve the linear models using Sims's (2002) algorithm
- Solve the nonlinear models using policy function iteration:
  - Use linear solution as an initial conjecture: $\tilde{c}^A(\mathbf{z}_t)$, $\pi^A(\mathbf{z}_t)$
  - For iteration i and node d, implement the following steps:
    1. Solve for $\{\tilde{w}_t, \tilde{y}_t, i_t^*, i_t\}$ given $\tilde{c}_{i-1}^A(\mathbf{z}_t^d)$ and $\pi_{i-1}^A(\mathbf{z}_t^d)$
    2. Use piecewise linear interpolation to solve for updated values of consumption and inflation, $\{\tilde{c}_{t+1}, \pi_{t+1}\}_{m=1}^M$, given each realization of the updated state vector, $\mathbf{z}_{t+1}$.
    3. Given $\{\tilde{c}_{t+1}, \pi_{t+1}\}_{m=1}^M$, solve for future output, $\{\tilde{y}_{t+1}^m\}_{m=1}^M$, which enters expectations. Then, numerically integrate.
    4. Use `csolve` to determine the values of the policy functions that best satisfy the equilibrium system
  - Define $\text{maxdist}_i \equiv \max\{|\tilde{c}_i^A - \tilde{c}_{i-1}^A|, |\pi_i^A - \pi_{i-1}^A|\}$. Continue iterating until $\text{maxdist}_i < 10^{-7}$ for all d.

Estimation Procedure

- Use quarterly data on per capita real GDP, the GDP price deflator, and the Fed Funds Rate from 1986Q1 to 2015Q4
- Use a Metropolis-Hastings algorithm with a particle filter to evaluate the likelihood of the posterior distribution
- Observation equation:

$$\begin{bmatrix} \log\left(\frac{RGDP_t/CNP_t}{RGDP_{t-1}/CNP_{t-1}}\right) \\ \log(DEF_t/DEF_{t-1}) \\ \log(1+FFR_t)/4 \end{bmatrix} = \begin{bmatrix} \log(g_t \tilde{y}_t^{gdp}/\tilde{y}_{t-1}^{gdp}) \\ \log(\pi_t) \\ \log(i_t) \end{bmatrix} + \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \\ \xi_{3t} \end{bmatrix},$$

  where $\xi \sim \mathbb{N}(0, \Sigma)$ is a vector of measurement errors.
- We adapt the particle filter to incorporate the information contained in the current observation, which helps the model better match outliers in the data (e.g., 2008Q4).

Adapted Particle Filter

1. Initialize the filter by drawing from the ergodic distribution.
2. For all particles $p \in \{1, \dots, N_p\}$ apply the following steps:
    2.1 Draw $\mathbf{e}_{t,p} \sim \mathbb{N}(\bar{\mathbf{e}}_t, I)$, where $\bar{\mathbf{e}}_t$ maximizes $p(\xi_t|\mathbf{z}_t)p(\mathbf{z}_t|\mathbf{z}_{t-1})$.

    2.2 Obtain $\mathbf{z}_{t,p}$, and the vector of variables, $\mathbf{w}_{t,p}$, given $\mathbf{z}_{t-1,p}$

    2.3 Calculate, $\xi_{t,p} = \hat{\mathbf{x}}_{t,p}^{model} - \hat{\mathbf{x}}_t^{data}$. The weight on particle p is

$$\omega_{t,p} = \frac{p(\xi_t|\mathbf{z}_{t,p})p(\mathbf{z}_{t,p}|\mathbf{z}_{t-1,p})}{g(\mathbf{z}_{t,p}|\mathbf{z}_{t-1,p}, \hat{\mathbf{x}}_t^{data})} \propto \frac{\exp(-\xi_{t,p}' H^{-1}\xi_{t,p}/2)\exp(-\mathbf{e}_{t,p}'\mathbf{e}_{t,p}/2)}{\exp(-(\mathbf{e}_{t,p} - \bar{\mathbf{e}}_t)'(\mathbf{e}_{t,p} - \bar{\mathbf{e}}_t)/2)}$$

    The model's likelihood at t is $\ell_t^{model} = \sum_{p=1}^{N_p} \omega_{t,p}/N_p$.

    2.4 Normalize the weights, $W_{t,p} = \omega_{t,p}/\sum_{p=1}^{N_p} \omega_{t,p}$. Then use systematic resampling with replacement from the particles.
3. Apply step 2 for $t \in \{1, \dots, T\}$. $\log \ell^{model} = \sum_{t=1}^{T} \log \ell_t^{model}$.

Particle Adaption

1. Given $\mathbf{z}_{t-1}$ and a guess for $\bar{\mathrm{e}}_t$, obtain $\mathbf{z}_t$ and $\mathbf{w}_{t,p}$.
2. Calculate $\hat{\mathbf{x}}_t^{model} = \left[\log(g_t\tilde{y}_t^{gdp}/\tilde{y}_{t-1}^{gdp}), \log(\pi_t), \log(i_t)\right]$.
3. Calculate $\xi_t = \hat{\mathbf{x}}_t^{model} - \hat{\mathbf{x}}_t^{data}$, which is multivariate normal:

$$p(\xi_t|\mathbf{z}_t) = (2\pi)^{-3/2}|H|^{-1/2}\exp(-\xi_t' H^{-1}\xi_t/2)$$
$$p(\mathbf{z}_t|\mathbf{z}_{t-1}) = (2\pi)^{-3/2}\exp(-\bar{\mathbf{e}}_t'\bar{\mathbf{e}}_t/2)$$

$H \equiv \mathrm{diag}(\sigma^2_{me,\hat{y}}, \sigma^2_{me,\pi}, \sigma^2_{me,i})$ is the ME covariance matrix.

4. Solve for the optimal $\bar{\mathrm{e}}_t$ to maximize

$$p(\xi_t|\mathbf{z}_t)p(\mathbf{z}_t|\mathbf{z}_{t-1}) \propto \exp(-\xi_t' H^{-1}\xi_t/2)\exp(-\bar{\mathbf{e}}_t'\bar{\mathbf{e}}_t/2)$$

We converted MATLAB's `fminsearch` routine to Fortran.

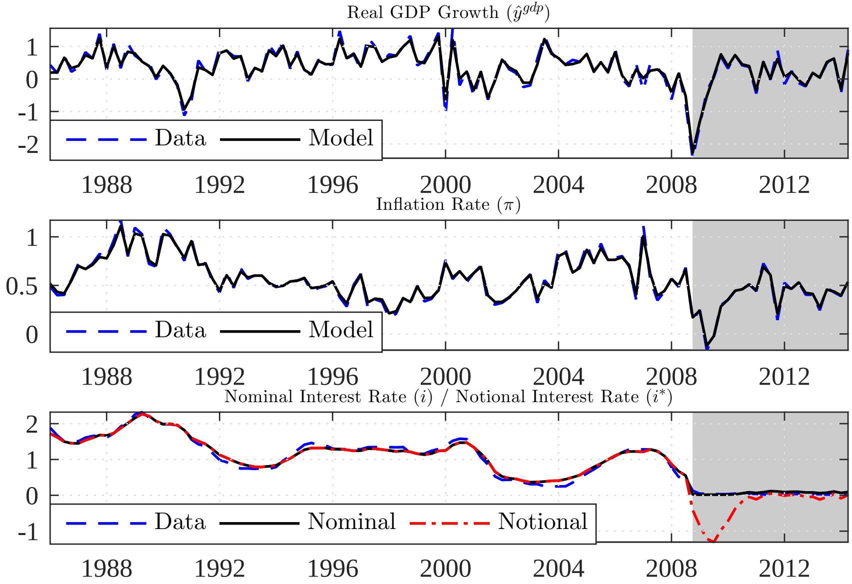
Calibrated Parameters

Steady-State Discount Factor	$\bar{\beta}$	0.9984
Frisch Elasticity of Labor Supply	$1/\eta$	3
Elasticity of Substitution between Goods	θ	6
Steady-State Labor	$\bar{n}$	0.33
Nominal Interest Rate Lower Bound	$\underline{\imath}$	1.00017
Real GDP Growth Rate Measurement Error SD	$\sigma_{me,\hat{y}}$	0.00194
Inflation Rate Measurement Error SD	$\sigma_{me,\pi}$	0.00075
Federal Funds Rate Measurement Error SD	$\sigma_{me,i}$	0.00206
Number of Particles	N_p	40,000
Number of Posterior Draws	N_d	100,000

Prior/Posterior Distributions

	Prior			Posterior		
Parameter	Distribution	Mean	SD	Mean	5%	95%
φ	Gam	80.000	20.000	96.80137	67.71867	131.85091
h	Beta	0.500	0.200	0.44428	0.30733	0.57745
ϕ_π	Norm	2.500	1.000	4.06383	3.33170	4.90267
ϕ_y	Norm	1.000	0.400	1.49057	1.12702	1.87727
$\bar{g}$	Norm	1.004	0.001	1.00376	1.00260	1.00489
$\bar{\pi}$	Norm	1.006	0.001	1.00622	1.00556	1.00683
ρ_g	Beta	0.500	0.200	0.20064	0.06547	0.37805
ρ_β	Beta	0.500	0.200	0.90245	0.87001	0.92958
ρ_i	Beta	0.500	0.200	0.81158	0.75375	0.86060
σ_ε	IGam	0.010	0.010	0.00968	0.00738	0.01241
σ_υ	IGam	0.010	0.010	0.00215	0.00159	0.00286
σ_ν	IGam	0.010	0.010	0.00199	0.00148	0.00261

Model Observables

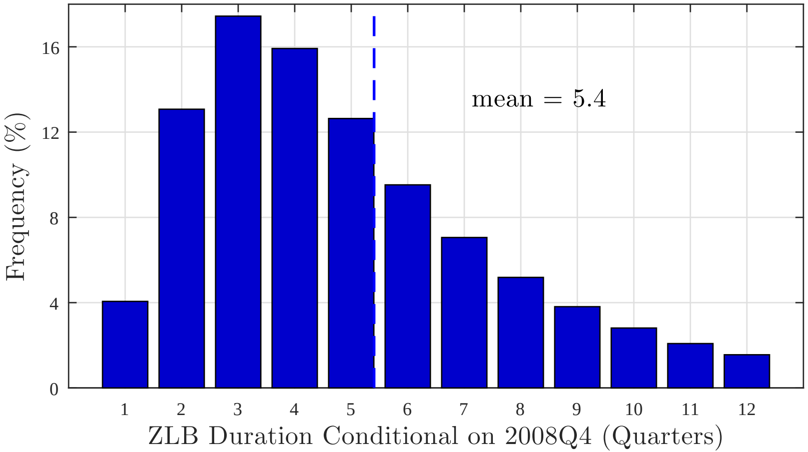


Empirical Fit of the Structural Model

	Real GDP Growth ($\hat{y}_t^{gdp}$)		Inflation Rate (π_t)		Interest Rate (i_t)	
	Mean	SD	Mean	SD	Mean	SD
Data	1.44	2.45	2.25	0.97	3.92	2.70
Model	1.54 $(0.66, 2.43)$	2.48 $(2.02, 3.04)$	2.50 $(1.99, 3.00)$	0.96 $(0.73, 1.25)$	4.70 $(3.35, 6.09)$	1.73 $(1.18, 2.40)$

	Autocorrelations			Cross-Correlations		
	$(\hat{y}_t^{gdp}, \hat{y}_{t-1}^{gdp})$	(π_t, π_{t-1})	(i_t, i_{t-1})	$(\hat{y}_t^{gdp}, \pi_t)$	$(\hat{y}_t^{gdp}, i_t)$	(π_t, i_t)
Data	0.30	0.63	0.99	0.01	0.18	0.47
Model	0.49 $(0.29, 0.66)$	0.72 $(0.60, 0.83)$	0.89 $(0.81, 0.95)$	-0.40 $(-0.63, -0.12)$	0.06 $(-0.22, 0.33)$	0.30 $(-0.04, 0.60)$

ZLB Duration Conditional on 2008Q4



Exogenous Uncertainty

- Suppose $x_{t+1} = \rho x_t + \sigma_{t+1}\epsilon_{t+1}$, $\epsilon \sim N(0,1)$
- σ follows an independent exogenous process
- Expected value of the forecast error

$$E_t[FE_{x,t+1}] = E_t\left[x_{t+1} - E_t x_{t+1}\right] = 0$$

- Expected volatility is

$$\sqrt{E_t[FE^2_{x,t+1}]} = \sqrt{E_t\left[(x_{t+1} - E_t x_{t+1})^2\right]} = \sqrt{E_t \sigma^2_{t+1}}$$

so σ determines the degree of uncertainty

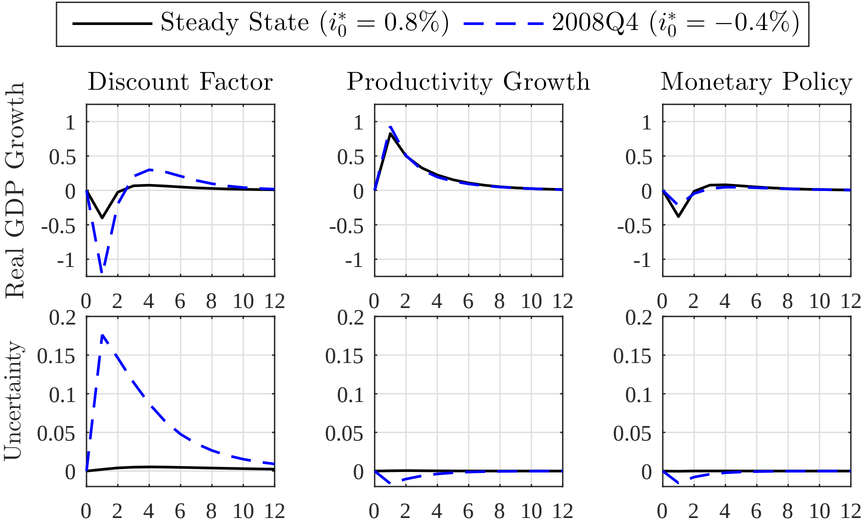
Endogenous Uncertainty

- Stochastic models contain uncertainty that is endogenous
- We quantify the degree of uncertainty that surrounds output (y) by following the logic of the SV literature
- The endogenous uncertainty surrounding real GDP is

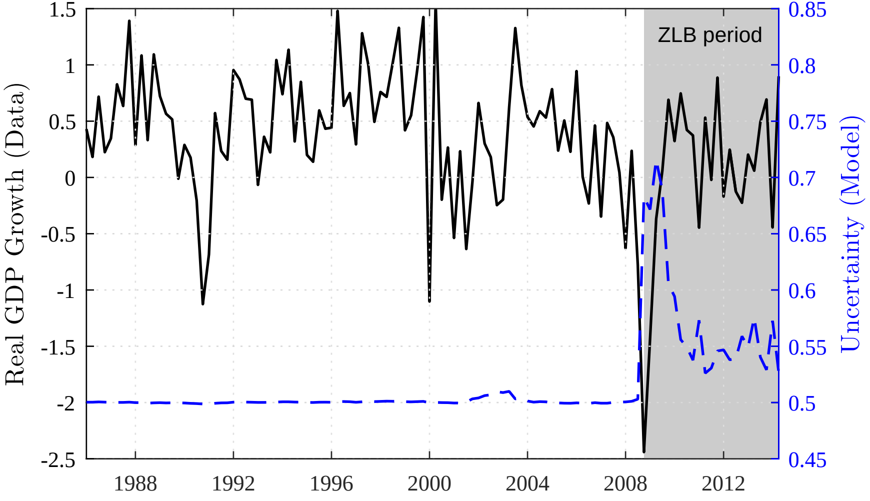
$$\sigma_{\hat{y}^{gdp},t} \equiv \sqrt{E_t[(\hat{y}^{gdp}_{t+1} - E_t\hat{y}^{gdp}_{t+1})^2]},$$

  which is the same measure of uncertainty JLN use.
- We focus on output uncertainty, but we also calculate this measure of uncertainty for other variables in the model.

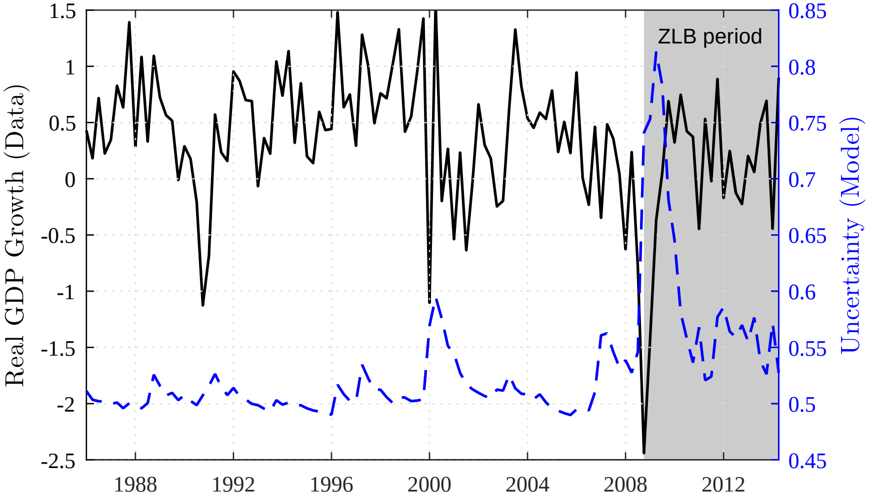
Generalized IRFs



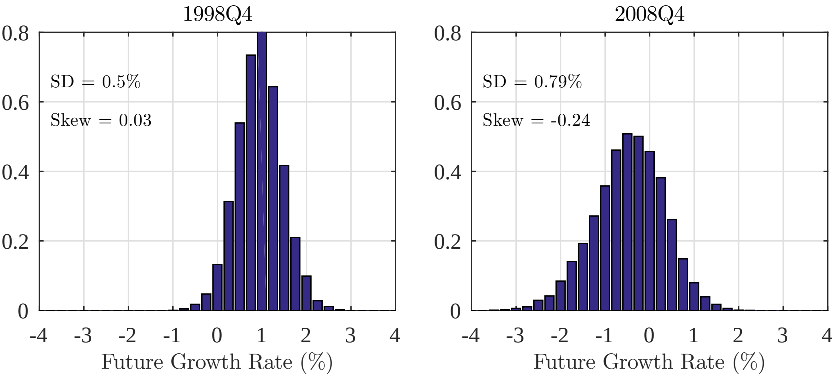
Excluding Stochastic Volatility



Including Stochastic Volatility



Real GDP Growth Forecasts (1-Quarter Ahead)



Real GDP Growth vs. Uncertainty

- Filter the model and generate a time series for real GDP growth uncertainty for each draw from the posterior
- Calculate the correlation between each filtered uncertainty series and per capita real GDP growth in the data

	Pre-ZLB Sample (1986Q1-2008Q3)	ZLB Sample (2008Q4-2014Q2)	Differences (ZLB - Pre-ZLB)
Excluding SV	–	−0.48***	–
Including SV	−0.19***	−0.54***	−0.34***

- Results are robust to removing 2008Q4 and 2009Q1

Additional Results & Supporting Evidence

Uncertainty Correlations

ZLB Sample \ Pre-ZLB Sample	DSGE	DSGE SV	VAR	VXO	BOS FD	SPF FD	JLN
DSGE	–	0.18	0.07	0.40	−0.35	0.08	0.05
DSGE SV	0.99	–	0.70	0.29	−0.03	−0.02	0.41
VAR	0.77	0.76	–	0.43	0.09	0.17	0.57
VXO	0.67	0.67	0.86	–	0.11	0.46	0.30
BOS FD	0.22	0.20	0.54	0.60	–	0.13	0.25
SPF FD	0.83	0.83	0.75	0.74	0.34	–	0.35
JLN	0.87	0.87	0.92	0.91	0.52	0.88	–

Pre-ZLB Cross Correlations $(\mathrm{corr}(\sigma_{y,t}, \hat{y}_{t+j}))$

	Output Leads				Uncertainty Leads		
	-3	-2	-1	0	1	2	3
DSGE	−0.17*	−0.14*	−0.07	0.02	0.07	0.14*	0.16*
DSGE SV	−0.13	−0.21**	−0.24**	−0.20**	−0.13	−0.26***	−0.29***
VAR	−0.18**	−0.28***	−0.29***	−0.35***	−0.31***	−0.29***	−0.26***
VXO	−0.07	0.03	−0.09	−0.09	−0.11	0.00	0.04
BOS FD	0.34***	0.17*	−0.11	−0.19**	−0.14*	−0.15*	−0.14*
SPF FD	−0.11	−0.17*	−0.24**	−0.12	−0.12	−0.09	−0.06
JLN	−0.16*	−0.31***	−0.35***	−0.45***	−0.41***	−0.43***	−0.45***

ZLB Cross Correlations
$(\mathrm{corr}(\sigma_{y,t}, \hat{y}_{t+j}))$

	Output Leads				Uncertainty Leads		
	-3	-2	-1	0	1	2	3
DSGE	−0.83***	−0.84***	−0.62***	−0.62***	−0.19	0.04	0.27
DSGE SV	−0.85***	−0.82***	−0.60***	−0.59***	−0.20	0.03	0.24
VAR	−0.54***	−0.67***	−0.79***	−0.79***	−0.55***	−0.16	0.13
VXO	−0.33*	−0.48**	−0.61***	−0.72***	−0.55***	−0.32*	−0.02
BOS FD	0.42**	0.07	−0.33*	−0.70***	−0.39**	−0.27	−0.15
SPF FD	−0.53***	−0.64***	−0.64***	−0.47***	−0.32*	0.03	0.12
JLN	−0.72***	−0.79***	−0.74***	−0.73***	−0.47**	−0.09	0.15

Inflation Uncertainty Correlations

	DSGE	DSGE SV	VAR	SPF FD CPI
Pre-ZLB Sample (1986Q1-2008Q3)	-0.23***	-0.28***	-0.19**	-0.25***
ZLB Sample (2008Q4-2014Q2)	-0.53***	-0.55***	-0.30	-0.55***
Difference (ZLB-Pre-ZLB)	-0.30***	-0.27***	-0.13	-0.31*

Conclusion

1. Data: Relationship between real GDP growth and uncertainty much stronger during the ZLB period
2. Theory: ZLB prevents the central bank from responding to adverse shocks, which increases macro uncertainty
3. Correlations between real GDP growth and uncertainty in the DSGE model have same key features as data
4. Results provide evidence ZLB is one important factor to consider when thinking about macro uncertainty