The Consequences of an Unknown Debt Target*

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January 18, 2015

ABSTRACT

Several proposals to reduce U.S. debt reveal large differences in their targets. We examine how an unknown debt target affects economic activity using a real business cycle model in which Bayesian households learn about a state-dependent debt target in an endogenous tax rule. Recent papers use stochastic volatility shocks to study fiscal uncertainty. In our setup, the fiscal rule is time-varying due to unknown changes in the debt target. Households infer the current debt target from a noisy tax rule and jointly estimate the transition probabilities. Three key findings emerge from our analysis: (1) Limited information about the debt target amplifies the effect of tax shocks through changes in expected tax rates; (2) The welfare losses are an order of magnitude larger when both the debt target state and transition matrix are unknown than when only the debt target state is unknown to households; (3) An unknown debt target likely reduced the stimulative effect of the ARRA and uncertainty about the sunset provision in the Bush tax cuts may have slowed the recovery and led to welfare losses.

Keywords: Bayesian learning; Limited information; Fiscal policy; Welfare; Anticipated utility
JEL Classifications: D83; E32; E62; H68

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1 INTRODUCTION

There is broad interest in understanding how fiscal uncertainty affects economic activity, especially given the recent fiscal turmoil around the world. Some of the recent literature examines the effects of fiscal uncertainty by looking at how exogenous changes in the volatility of fiscal instruments affect various macroeconomic outcomes [e.g., Fernández-Villaverde et al. (2013), Born and Pfeifer (2014)]. We take a different approach. In our model, the mean, rather than the volatility, of government debt is time-varying. Households are uncertain about whether short-run changes in tax policy imply a change in the long-run level of debt or whether future policy will return debt to its current long-run level, which captures the uncertainty inherent to the political process. This type of uncertainty matters because it obscures whether changes in taxes are temporary or permanent.

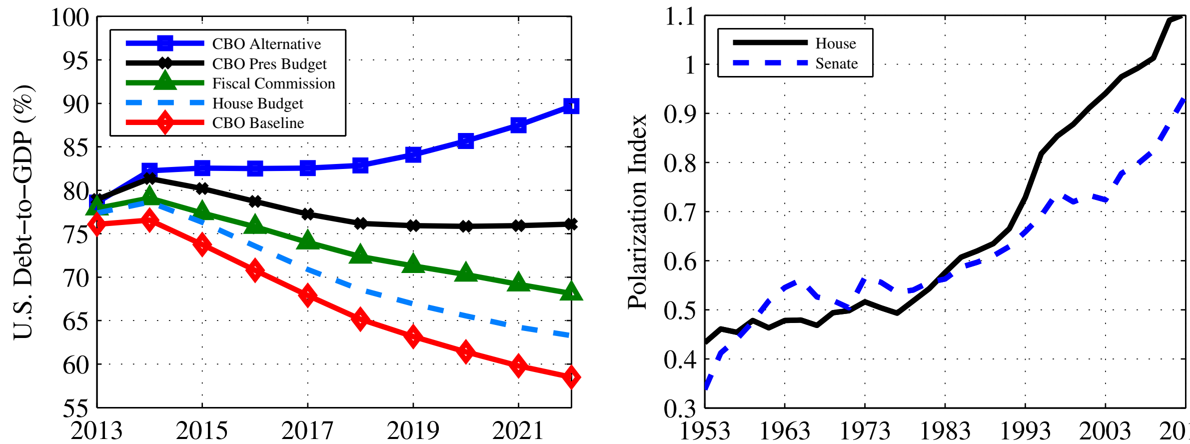
We focus on the uncertainty surrounding the long-run level of debt in the U.S. because the role of government is typically measured by the size of the national debt and/or the cost of servicing it. Positions within that debate are compared by their long-run outcomes, which correspond to very different future fiscal policies. Thus, the effects of uncertainty about the level of debt and the first moments of the implied policy changes are paramount to fluctuations in their second order moments, which is the focus of the stochastic volatility approach. Moreover, Mumtaz and Surico (2013) find that uncertainty about debt sustainability has had a larger and more significant effect on economic activity than uncertainty about monetary, tax, and government spending policies.[1]

The American Recovery and Reinvestment Act of 2009 (ARRA) and the Economic Growth and Tax Relief Reconciliation Act of 2001 (known as the Bush tax cuts) provide two recent examples of the type of uncertainty we are trying to capture. The ARRA was a combination of temporary tax cuts and spending increases that raised the national debt. When the policy was passed, households did not know whether the tax cuts would permanently increase the national debt or whether future policy would adjust to maintain the current level of debt. The Bush tax cuts were passed in 2001, extended for 2 years in 2011, and then permanently extended in January 2013. Despite bipartisan support for extending most of the tax cuts, political gridlock meant that people did not know which tax cuts, if any, Congress would extend. Low tax rates contribute to a higher national debt and people could interpret a permanent extension as a change in the long-run level of debt. Allowing government debt to rise to a higher long-run level in either of these examples would lower expected future taxes. Since households do not know whether Congress will enact reforms to reduce the debt-to-GDP percentage or follow current policy, they face uncertainty about future tax rates.

Households are always uncertain about discretionary policy changes, but the uncertainty surrounding changes in the long-run level of debt has risen because U.S. public debt recently surpassed 70% of GDP, which is well above its post-WWII average of 45%. The responses to the Great Recession demonstrate the lack of consensus about the conduct of fiscal policy. As an example, the ARRA was immediately followed by a debate about deficit reduction, which resulted in the Budget Control Act of 2011. While several countries have fiscal rules, including specific targets for spending, taxes, and debt as a percent of GDP, the U.S. replaced its deficit target in 1990 with informal pay-as-you-go (PAYGO) requirements that are easily waived by members of Congress.[2]

[1] There is also ample evidence people are concerned with future debt in the U.S. For example, an April 2011 Pew Research poll found 80% of Americans thought the federal budget deficit is a major problem the country must address now, which was up from 70% in their December poll. A March 2014 Gallup poll found 80% of Americans worried about federal spending and the budget deficit either a great deal or a fair amount. Earlier polls reported similar views.

[2] The 2011 revisions to the Stability and Growth Pact of the European Union set a debt target equal to 60% of GDP



(a) Long-run debt targets implied by some of the U.S. debt reduction proposals, compared with the baseline (current law) and alternative fiscal financing (current policy) scenarios in the CBO's 2012 Long-Term Budget Outlook. Source: Committee for a Responsible Federal Budget

(b) Index of polarization of the U.S. Congress, measuring the difference between the two parties' positions on a conservative-liberal interval, $[-1, 1]$. The minimum of this index is 0 (full agreement) and the maximum is 2 (full polarization). Source: Poole and Rosenthal (2012)

Figure 1: Sources of limited information about the debt target.

There are several proposals to reduce the level of debt in the U.S., but their implied 10-year targets are very different, with estimates ranging from 63 to 76 percent of GDP in 2022 (figure 1a).[3] The projections by the CBO and other independent agencies over horizons beyond 10 years are even more diffuse, but we focus on the projections made over a 10-year time frame because they correspond to the federal budget window. We view the long-run levels of debt implied by the various proposals as implicit debt targets. With political polarization at a record level (figure 1b), it is possible that none of the proposals pass Congress in the next 10 years and the debt-to-GDP percentage continues to rise, as the Congressional Budget Office (CBO) currently projects.[4] The differences in these outcomes demonstrate that the debt target is unknown to households.

In our model, changes in taxes may be temporary due to discretionary tax shocks or persistent due to a change in the long-run level of debt, which we refer to as a debt target. Households do not observe the debt target or know whether changes in the tax rate are the result of discretionary policy. Each year households use their observations of the tax rate and debt-to-GDP ratio to infer whether the debt target has changed—which affects their expectations of future tax policy—or whether the observed changes in tax policy are temporary so their expectations remain unchanged.

for all member nations. If a country exceeds that limit, it is required to reduce its excess debt (as a percent of GDP) by one-twentieth each year to avoid sanctions. The 2010 Budget Act in Sweden mandates a net lending target equal to 1% of GDP. In New Zealand, the Fiscal Responsibility Act requires a "prudent" debt level, and the current goal is to reduce debt to 20% of GDP by 2020. Canada recently committed to a balanced budget by 2015 and set a debt-to-GDP target of 25% by 2021. The Gramm-Rudman-Hollings Balanced Budget Act of 1985 established formal deficit targets in the U.S., but they were replaced with a less restrictive PAYGO system in 1990. That system was abandoned in 2002. Although it was reestablished in 2007, it now includes an emergency exemption that makes it easier to circumvent.

[3]The International Monetary Fund (2013) provides a detailed overview of the fiscal outlook in advanced and emerging economies. They remark that many advanced economies "face a lengthy, difficult, and uncertain path to fiscal sustainability," despite the decline in global deficits. They further contend that a clear and credible plan that guides future policy is essential to fiscal sustainability, and the lack of such plans in the U.S. and Japan is a concern.

[4]Alesina and Tabellini (1990) find that a higher degree of polarization leads to higher equilibrium debt in a model where control of the government alternates between two political parties with different preferences for public goods.

Households also do not know how the debt target evolves over time since it follows a Markov process with an unobserved transition matrix. Thus, they form expectations about future tax rates conditional on their estimate of the transition matrix, which is based on their past observations.[5]

We endow households with alternative information sets to isolate the sources of uncertainty. Our benchmark is the full information set, where current and past debt targets and the transition matrix are known to households. We compare two limited information cases to that benchmark: (1) current and future debt targets are unknown and the transition matrix is known, and (2) both the debt target and transition matrix are always unknown. The unknown debt target reflects that there is no formal legislation that requires Congress to commit to a future debt target while the unknown transition matrix accounts for political uncertainty. For example, a wave election could install a new governing party that decides to implement legislation that changes the future path of debt. Or, last minute policy changes that result in short-run changes to taxes may contradict certain long-run goals. Thus, we view the full information case as a situation where Congress establishes a committee that publicly announces a debt target and is insulated from the political process.[6]

When households incorrectly infer the current debt target, their decision rules differ from the rules based on the actual debt target. If the transition matrix is also unknown, the accuracy of their inferences about the debt target depend on the estimates of the transition matrix. Learning about those two aspects of fiscal policy affects households' decisions and influences future tax rates, which endogenously respond to debt. Since the tax rate is levied against each household's income, the limited information affects the real economy. Three key findings emerge from our analysis:

1. Limited information about the debt target amplifies the effect of tax shocks through changes in expected taxes, which have a cumulative effect on the capital stock. The unknown debt target causes high frequency deviations from the full information case, while the unknown transition matrix causes lower frequency deviations that are larger in magnitude.

2. Our results reinforce the popular viewpoint that fiscal uncertainty reduces welfare.[7] There are losses in the median of the welfare distributions and the losses are larger than the gains in the tails of the distributions in both limited information cases. Also, the median welfare losses are an order of magnitude larger in Case 2 than in Case 1.

3. An unknown debt target likely reduced the stimulative effect of the ARRA. We find that the uncertainty about whether debt would rise to a new long-run level or whether future policy would adjust to maintain the current debt level led to lower output and investment. Moreover,

[5]We chose to simplify the model by specifying that only distortionary taxes finance debt. Most proposals call for a balanced approach of higher revenues and lower spending. In an August 2011 Gallup poll, 60% of respondents favored increasing tax revenues with tax reform and 66% favored raising income tax rates on upper-income households. Only 42% favored changes to Social Security and Medicare. In a November 2012 Gallup poll, 86% of respondents wanted Congress to achieve deficit reduction with some tax increases. In a 2013 survey, the National Small Business Association found 82% of small businesses support deficit reduction through higher revenues. Since the composition of future financing is also unknown, we view our results as a floor on the consequences of an unknown debt target.

[6]Kirsanova et al. (2007) propose an independent fiscal policy council, similar to the CBO, that recommends sustainable fiscal policies and requires a response from the government as to whether the recommendations will be adopted. Since monetary and fiscal policy can have similar effects on the economy, Leeper (2009) advocates for the transformation of fiscal policy institutions in a similar way as countries have transformed their monetary policy objectives.

[7]See, for example, *The Wall Street Journal*, "Want Growth? Try Stable Tax Policy" (December 21, 2011), *The Economist*, "The cloud of uncertainty. Dithering in the dark." (June 16, 2012) and "Pointless, painful uncertainty" (July 18, 2012), and *The New York Times*, "Uncertainty Is the Enemy of Recovery" (April 28, 2013).

uncertainty about the sunset provision in the Bush tax cuts may have slowed the recovery and led to welfare losses. If, however, the debt target is revealed, even after a period of limited information, then capital, output, and welfare increase and the economy quickly recovers.[8]

The results in the stochastic volatility (SV) literature are similar to our finding in Case 1, even though we focus on a time-varying debt target and not time-varying volatility. For example, Fernández-Villaverde et al. (2013) and Born and Pfeifer (2014) find a two standard deviation volatility shock to various fiscal instruments decreases output by 0.11% and 0.045%. In Case 2, when the transition matrix is also unknown, the limited information often generates a larger effect.

There are important differences between our approach to modeling fiscal uncertainty and the SV approach. As an example, suppose a model includes a tax shock. SV is introduced by assuming the standard deviation of the shock is time-varying. In our setup, the fiscal rule is time-varying due to changes in the debt target that can and do occur over time. The two approaches to modeling fiscal uncertainty also have different effects on households' behavior. Changes in the volatility of the tax shock alter the expected volatility of the forecast error for future taxes but have no effect on households' expectations functions. When the debt target changes, however, any parameter that is a function of the current debt target also changes, which alters households' expectations functions.

Several early papers that study fiscal policy in a stochastic general equilibrium model allow for discrete shifts in policy, so that households expect recurring regime changes [Aizenman and Marion (1993); Bizer and Judd (1989); Dotsey (1990)]. A key feature in more recent papers is that the current regime is unobserved [Andolfatto and Gomme (2003); Bianchi and Melosi (2013); Davig (2004); Leeper and Zha (2003); Schorfheide (2005)]. Households rationally form expectations about the regime using Bayesian updating. The closest paper to ours is Davig (2004). Davig analyzes tax reform in a model where a tax rule is a function of the debt-to-output ratio, which evolves according to a hidden state Markov chain. If households place a high probability on a change in the tax rule, then they expect the tax reform to persist. That possibility can alter the sign of the tax elasticity of investment. We build on this literature by also removing the transition probabilities from households' information sets. As far as we know, ours is one of only two papers that study how a hidden transition matrix affects household behavior in a general equilibrium model.[9]

In our model, households rationally learn about the unknown debt target and use observations of the tax rate and debt-to-output ratio to adaptively learn the transition matrix. The least squares learning literature uses a similar approach [see Evans and Honkapohja (2001, 2009) for an overview]. Contrary to rational expectations, households do not have full information about the structure of the economy and act as econometricians using historical data to make inferences about unknown components. For example, Giannitsarou (2006) finds that when households have limited information about the structure of the economy, a capital tax cut is ineffective during a recession. More recently, Mitra et al. (2012, 2013) analyze policy changes in the context of a real business cycle model. Mitra et al. (2012) report output multipliers under learning, and Mitra et al. (2013) find that anticipated changes to lump-sum taxes under learning lead to large impact effects and gradual hump-shaped impulse responses. In our model, households know the structure of the economy. They filter a noisy signal using the correct model and rationally form expectations about

[8] In a related analysis, House and Shapiro (2006) find the timing of the Bush tax cuts reduced output, labor, and investment. The phase-in of the Bush tax cuts partly resulted in a slow recovery from the 2001 recession, but the economy rebounded when the phase-ins were removed in the Jobs and Growth Tax Relief Reconciliation Act of 2003.

[9] In an analysis of the 2008 U.S. credit crisis, Boz and Mendoza (2014) use a model with a collateral constraint where agents do not observe the transition probabilities governing periods of high- versus low-borrowing ability.

the debt target. However, they use an anticipated utility approach [Kreps (1998)] to form expectations conditional on their estimate of the transition matrix. This means households adaptively form expectations, conditional on a history of observations. Hollmayr and Matthes (2013) also use an anticipated utility approach, but they focus on limited information about government spending.

Another source of limited information is the resolution to the looming unfunded liabilities in the U.S. A segment of the rational expectations literature assumes households form expectations over a range of potential outcomes [Bi et al. (2013); Davig and Foerster (2014); Davig and Leeper (2011); Davig et al. (2010, 2011); Richter (2014)]. This work shows that expectations about future policy, and the outcomes households condition on, significantly affect current economic activity. However, households know the current policy regime and the true probability distributions.

Papers in the deficit bias literature share a similar motivation. That literature explains systematic deficits by modeling political interest groups that bargain over conflicting goals. Alesina and Tabellini (1990) show there is a bias toward budget deficits in a model where control of the government alternates between two political parties with different preferences for public goods. von Hagen and Harden (1995) find that if some budget administrators place a greater emphasis on the collective interest, rather than specific constituencies, the government can overcome fiscal illusion—the tendency for politicians to pursue projects that have a large net benefit to their constituency at the expense of all taxpayers. Since the long-run level of debt in our model can both increase and decrease, debt target uncertainty captures the possibility of deficit bias as well as the possibility that Congress implements fiscal policies that lead to the eventual reduction of debt.

Finally, it is important to note that fiscal uncertainty is one component of the broader concept of economic uncertainty. Many people have argued that uncertainty about the recovery has reduced demand and prolonged the recession. The stochastic volatility literature studies this topic by examining how uncertainty about productivity and the business cycle affects the economy. That literature is mixed on the impact of volatility shocks. Some papers find that higher volatility reduces output [Alexopoulos and Cohen (2009); Basu and Bundick (2012); Bloom (2009); Bloom et al. (2014)], while others report little or no impact [Bachmann and Bayer (2013); Chugh (2013)].

The rest of the paper is organized as follows. Section 2 lays out the model, information sets, and solution method. Section 3 describes the calibration and connects the model to the data. Section 4 demonstrates that limited information amplifies the effects of tax shocks using impulses responses and model simulations. Section 5 shows the welfare costs of the limited information. Section 6 examines the effects of the uncertainty surrounding recent U.S. legislation. Section 7 concludes.

2 Economic Model, Expectations, and Numerical Methods

We use a real business cycle model to study the consequences of an unknown debt target.[10] Our innovation is to introduce an expectations operator conditional on households' inference about the current debt target state and its transition matrix. The benchmark case, defined by the full information set, is when households know current and past debt targets and the transition matrix. We compare this case with households' behavior conditional on two limited information sets.

2.1 Model

A unit measure of households choose $\{c_j, n_j, i_j, b_j\}_{j=t}^{\infty}$ to maximize expected lifetime utility, given by, $\mathbb{E}_t^{\ell} \sum_{j=t}^{\infty} \beta^{j-t}[\log c_j - \chi n_j^{1+\eta}/(1+\eta)]$, where $\beta \in (0,1)$ is the subjective

[10] Basu and Bundick (2012) and Fernández-Villaverde et al. (2013) show sticky prices are necessary for volatility shocks to generate business cycle fluctuations. We use a time-varying mean, so uncertainty will have real effects.

discount factor, $1/\eta$ is the Frisch elasticity of labor supply, c is consumption, n is labor hours, i is investment, b is a one-period real government bond, and $\mathbb{E}_t^\ell$ is an expectation operator conditional on information set ℓ (defined in section 2.2). Each household's choices are constrained by

$$c_t + i_t + b_t = (1-\tau_t)(w_t n_t + r_t^k k_{t-1}) + r_{t-1} b_{t-1} + \bar{z}, \tag{1}$$
$$k_t = i_t + (1-\delta) k_{t-1}, \tag{2}$$

where w is the real wage, r^k is the real rental rate of capital, k is the capital stock, which depreciates at rate δ, r is the real interest rate, $\bar{z}$ is a fixed government transfer, and τ is the tax rate on income.

A perfectly competitive firm produces output according to $y_t = \bar{a} k_{t-1}^{\alpha} n_t^{1-\alpha}$, where $\bar{a}$ is technology and $\alpha \in (0,1)$. Each period the firm chooses its capital and labor to maximize profits.

The fiscal authority finances a constant level of spending, $\bar{g}$, and transfers, $\bar{z}$, by levying taxes on income and issuing one-period debt. The government's flow budget constraint is given by

$$b_t + \tau_t(w_t n_t + r_t^k k_{t-1}) = r_{t-1} b_{t-1} + \bar{g} + \bar{z}. \tag{3}$$

In most models, the debt target is fixed and it is usually calibrated to the average debt-to-GDP ratio. With that specification, tax policy adjusts to return debt to its average level. In reality, the debt target is typically unknown. We modify the tax rule so the tax rate endogenously responds to deviations of the debt-to-output ratio from a hidden (state-dependent) debt target according to

$$\tau_t = \bar{\tau}(s_t) + \gamma(b_{t-1}/y_{t-1} - \overline{by}(s_t)) + \varepsilon_t, \tag{4}$$

which reflects that the fiscal policy rule can and does change. The debt target, $\overline{by}(s_t)$, evolves according to an m-state Markov process with transition matrix, P. For row i and column j of P, element $p_{ij} = \Pr\{s_t = j | s_{t-1} = i\}$ for $i, j \in \{1, \ldots, m\}$, where $0 \leq p_{ij} < 1$ and $\sum_{j=1}^{m} p_{ij} = 1$ for all i. Tax policy then adjusts to bring debt in line with the current debt target state.[11] $\bar{\tau}(s_t)$ is a state dependent intercept that is consistent with the current debt target state. The tax shock, $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, is a proxy for discretionary tax policy (e.g., tax cuts in recessions). When the debt target state is unknown, households infer the state with their observation of $x_t \equiv \tau_t - \gamma b_{t-1}/y_{t-1}$.[12]

The aggregate resource constraint is given by $c_t + i_t + \bar{g} = y_t$. A competitive equilibrium consists of infinite sequences of quantities, $\{c_j, n_j, k_j, i_j, y_j\}_{j=t}^{\infty}$, prices, $\{w_j, r_j^k, r_j\}_{j=t}^{\infty}$, government policies, $\{\tau_j, b_j\}_{j=t}^{\infty}$, and shocks, $\{\varepsilon_j, s_j\}_{j=t}^{\infty}$ that satisfy the law of motion for capital, the household's and firm's optimality conditions, the government's budget constraint, the fiscal policy rule, the aggregate resource constraint, and the transversality conditions for capital and bonds.

2.2 Expectations Formation

The alternative information sets are summarized in table 1. To formally describe each household's information sets, we write the model compactly as

$$\mathbb{E}\left[f(\mathbf{v}_{t+1}, \mathbf{z}_{t+1}^\ell, \mathbf{v}_t, \mathbf{z}_t^\ell) | \Omega_t^\ell\right] = 0,$$

[11] There is a lack of consensus about the fiscal policy rule. However, a tax rule that responds to government debt is common. It is supported, in part, by the empirical findings of Bohn (1998), who shows the U.S. primary surplus increased with the debt-to-GDP ratio. Davig and Leeper (2006), and references therein, provide further evidence.

[12] We focus on an unknown debt target instead of uncertainty about how strongly taxes respond to government debt for two main reasons. First, while a quick adjustment to a new debt target would imply a large change in future tax rates, we speculate that these adjustments would be dwarfed by potentially large changes in the debt target. Second, households' beliefs about the debt target can alter the expected direction of future changes in tax rates, whereas uncertainty about the speed of the transition to a new (known) debt target only affects the magnitude of those changes.

where f is a vector-valued function that represents the equilibrium system of equations, $\mathbf{v}_t \equiv (c_t, n_t, k_t, i_t, b_t)$ is a vector of control variables,

$$\mathbf{z}_t^\ell \equiv \begin{cases} (k_{t-1}, r_{t-1}b_{t-1}, \tau_t, s_t), & \text{for } \ell = 0, \\ (k_{t-1}, r_{t-1}b_{t-1}, \tau_t, \mathbf{q}_t), & \text{for } \ell \in \{1,2\}, \end{cases}$$

is a vector of state variables, and Ω_t^ℓ is the information set in Case $\ell \in \{0,1,2\}$. $\mathbf{q}_t$ is a vector of probabilities that $s_t = i$, where $i \in \{1,\ldots,m\}$. The information sets are defined as:

$$\Omega_t^0 \equiv \{M, \Theta, \mathbf{z}_t^0, P\}, \qquad \Omega_t^1 \equiv \{M, \Theta, \mathbf{z}_t^1, P\}, \qquad \Omega_t^2 \equiv \{M, \Theta, \mathbf{z}_t^2, \hat{P}_t\},$$

where M is the structural model, $\Theta \equiv (\beta, \eta, \chi, \delta, \bar{a}, \alpha, \gamma, \{\bar{\tau}(i)\}_{i=1}^m, \{\overline{by}(i)\}_{i=1}^m, \sigma_\varepsilon^2)$ is a vector of model parameters, and P is the transition matrix. $\hat{P}_t$ is an estimate of the transition matrix, which is conditional on the history of observations denoted by $\mathbf{x}^t \equiv \{x_j\}_{j=0}^t$.

	Full Information	Limited Information	
	Case 0	Case 1	Case 2
Current Debt Target State	Known	Unknown	Unknown
Debt Target Transition Matrix	Known	Known	Unknown

Table 1: Alternative information sets

Expectations are formed in one of two ways, depending on the information set. In Case 0, households know the debt target state. Given $s_t = i$, their expectations formation is given by

$$\mathbb{E}\left[f(\mathbf{v}_{t+1}, \mathbf{z}_{t+1}^0, \mathbf{v}_t, \mathbf{z}_t^0)|\Omega_t^0\right] = \sum_{j=1}^m p_{ij} \int_{-\infty}^{+\infty} f(\mathbf{v}_{t+1}, \mathbf{z}_{t+1}^0, \mathbf{v}_t, \mathbf{z}_t^0)\phi(\varepsilon_{t+1})\mathrm{d}\varepsilon_{t+1} = 0, \tag{5}$$

where p_{ij} is the true probability of transitioning from $s_t = i$ to $s_{t+1} = j$ and $\phi(\cdot)$ is the normal probability density function. Note that $\tau_{t+1} \in \mathbf{z}_{t+1}^0$ is a function of the future tax shock, ε_{t+1}.

In Cases 1 and 2, tax shocks obscure the debt target and force households to make inferences about the state. After updating the tax rate (given the debt-to-output ratio, and realizations of s_{t+1} and ε_{t+1}), households compute the possible future observations, $x_{t+1} \equiv \tau_{t+1} - \gamma b_t/y_t = \bar{\tau}(s_{t+1}) - \gamma\overline{by}(s_{t+1}) + \varepsilon_{t+1}$, which have a mixed probability density of m normal distributions. In other words, households estimate the updated vector of conditional probabilities, $\mathbf{q}_{t+1} = \Pr(s_{t+1} = j|\mathbf{x}^{t+1})$, with the possible observations of x_{t+1}. Their expectation formation is given by

$$\mathbb{E}\left[f(\mathbf{v}_{t+1}, \mathbf{z}_{t+1}^\ell, \mathbf{v}_t, \mathbf{z}_t^\ell)|\Omega_t^\ell\right] = \sum_{i=1}^m \mathbf{q}_t(i) \sum_{j=1}^m p_{ij}^\ell \int_{-\infty}^{+\infty} f(\mathbf{v}_{t+1}, \mathbf{z}_{t+1}^\ell, \mathbf{v}_t, \mathbf{z}_t^\ell)\phi(\varepsilon_{t+1})\mathrm{d}\varepsilon_{t+1} = 0, \tag{6}$$

where $\ell \in \{1,2\}$, $p_{ij}^1 \in P$ is a true transition probability, and $p_{ij}^2 \in \hat{P}_t$ is an estimated probability.

2.3 Algorithm and Simulation Procedure

The model contains discrete changes in the debt target, which require us to solve the fully nonlinear model. We use a fixed-point projection method with linear interpolation and Gauss-Hermite quadrature. This solution technique simultaneously solves for the optimal decision rules at each point in the discretized state space. More specifically, it assumes the current decision rules hold at time t and $t+1$ and uses the Euler equations to back out the updated decision rules. In our model, the number of states, $m > 2$, so the restriction $\sum_{i=1}^{m} \mathbf{q}_t(i) = 1$ imposes a constraint on the discretized state space. See appendices B.1 and B.2 for details on how we implement the solution procedure and discretize the state space.[13]

In all cases, we initialize the simulation procedure by drawing a random sequence of T true debt target states, $\{s_t\}_{t=0}^{T}$, and discretionary tax shocks, $\{\varepsilon_t\}_{t=0}^{T}$. Each period the fiscal authority sets the tax rate with (4), given the debt-to-output ratio, the debt target state, and the discretionary tax shock. The current tax rate is always observed by households. Simulating the model in Case 0 is straightforward, given the solution where expectations are evaluated according to (5).

In Case 1, households use the true transition matrix and their inference about the state is built into their expectations in (6). Simulating Case 1 requires updating each household's inference of the state, $\mathbf{q}_t$, conditional on $\mathbf{x}^t$ each period using the nonlinear filter in Hamilton (1989) (see appendix C.1 for details). Thus, households learn about the debt target state as time evolves.

In Case 2, households estimate the transition matrix each period, but, in expectation, that estimate is time invariant. In this sense, we follow the anticipated utility approach of Kreps (1998).[14] Under Bayesian learning, households know their estimates may differ from the true transition matrix, but with anticipated utility they form expectations as if their estimates are the truth. They use a Gibbs sampler [Albert and Chib (1993)] to draw a history of debt target states conditional on their observations, $\mathbf{x}^t$, and then draw the transition matrix given that history.[15] Their estimate of the transition matrix, $\hat{P}$, is the average of the last half of the resulting chain (see appendix C.2 for details). Each period households re-optimize after updating their estimates of the transition matrix. Households use Bayesian methods to update their beliefs about the debt target state and transition matrix and an anticipated utility approach to form expectations.

In all cases, the fiscal authority uses the updated debt-to-output ratio, which is based on each household's decisions, the true debt target state, and the discretionary tax shock, to set the tax rate.

3 CALIBRATION AND DATA CONNECTIONS

We calibrate the model at an annual frequency to study limited information over multiple years. The discount factor, β, is set to 0.9615, which corresponds to a 4% real interest rate. The depreciation rate, δ, is set to 10% and the cost share of capital, α, is set to 0.33. The Frisch labor supply elasticity, $1/\eta$, is set to 0.5, which is consistent with the value used by the CBO and Chetty et al. (2012). The leisure preference parameter, χ, is calibrated so that steady-state labor equals $1/3$ of the available time in the mid debt target state. The level of technology, $\bar{a}$, is set so that steady-state output is equal to 1 in the mid debt target state. We fix χ and $\bar{a}$ in the other states and solve for the implied steady-state values of taxes, labor, and output that are consistent with the debt target.

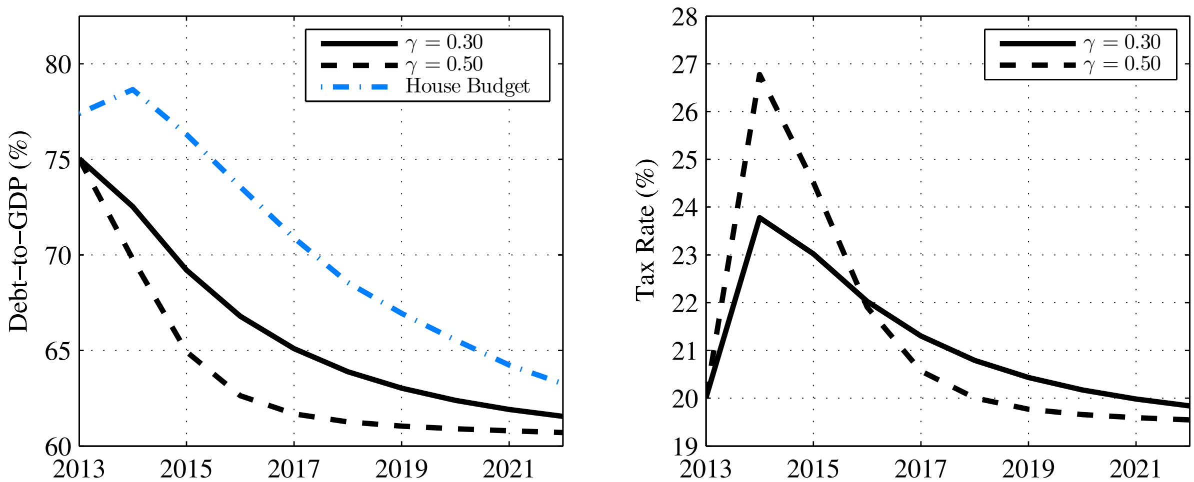
Government spending and transfers are set to 8% and 9% of output, which matches their averages from 1947-2013. There are three debt target states: low (60%), mid (75%), and high (90%). Those values correspond to the House Republican's 2012 deficit reduction proposal, the President's 2012 budget, and the CBO's alternative fiscal scenario. We include 3 states so the distance

[13] See Judd (1998) and Richter et al. (2014) for examples of fixed-point projection methods. The advantage of this procedure over time iteration is that it does not require a nonlinear solver to solve for the decision rules on each node.

[14] Including the entire transition matrix distribution in the state is computationally infeasible. However, Cogley and Sargent (2008) show the anticipated utility approach provides a good approximation for Bayesian decision making.

[15] We use a Gibbs sampler because maximum likelihood estimation does not perform well with a short sample.

between the debt targets imply plausible adjustments to the tax rate in the short-run. Given the upper and lower bound on the 10-year projections of the debt-to-GDP ratio (figure 1a), only including 2 states would imply large changes to the tax rate. Moreover, with 3 states the mid debt target is unique from the high and low debt targets in that households may expect an increase or decrease in the debt target. The response of the tax rate to changes in the debt-to-output ratio, γ, is set to 0.3 so a switch in the true debt target state takes the economy about 10 years to adjust to its new long-run equilibrium in Case 0, which is consistent with Congress's budget window. It also guarantees a sufficient response by the fiscal authority to ensure stable long-run debt dynamics.



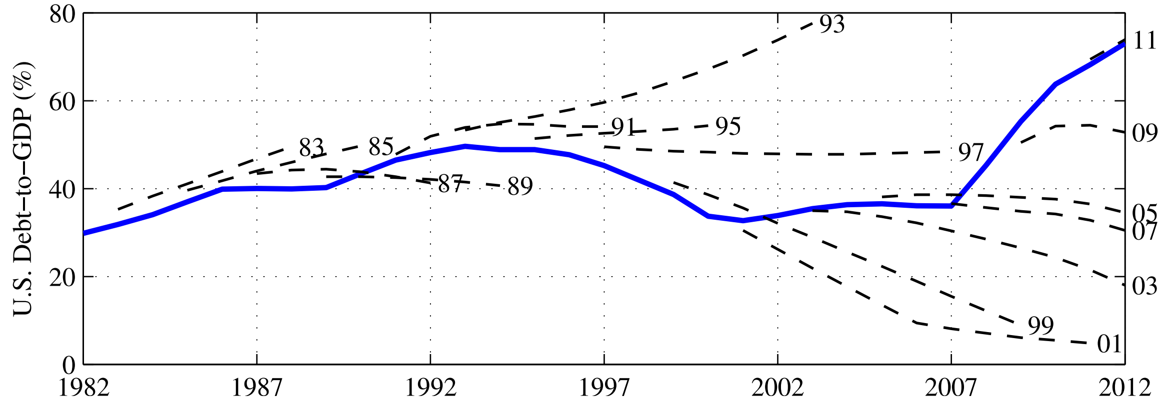
(a) Path to the new long-run debt-to-GDP ratio

(b) Path to the new long-run tax rate

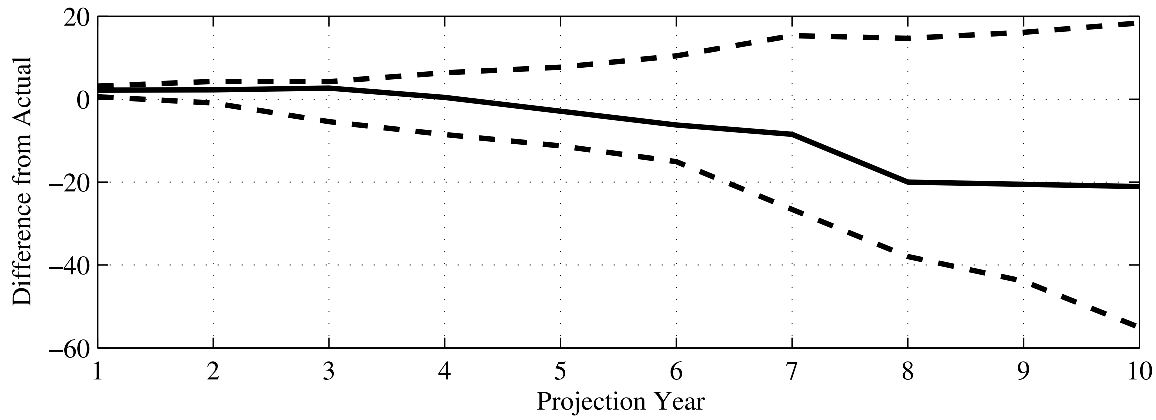
Figure 2: Full information paths after a switch from the mid (75%) to low (60%) debt target.

Figure 2a shows the speed of convergence of the debt-to-output percentage under full information after a switch in the debt target state from mid (75%) to low (60%). The 2012 House proposal is provided for comparison. When $\gamma = 0.3$, the speed of convergence matches closely with the House's 10-year proposal. The implied 10-year target also matches closely. In the model, the debt target is approached asymptotically, so there is a trade-off between matching the 10-year target and the speed of convergence. We further justify our choice of γ by at looking the short-run adjustment in the tax rate implied by the endogenous tax rule. A 3.75% short-run adjustment (figure 2b) seems consistent with current deficit reduction proposals, considering the 2012 American Taxpayer Relief Act changed marginal rates by similar amounts (e.g., the top marginal income tax rate rose by 4.6 and the payroll tax rose by 2 percentage points) but not enough to achieve the low debt target.

To obtain a historical sense for how frequently policy changes over time, figure 3a plots each of the CBO's baseline projections of the debt-to-GDP percentage (dashed lines) with the actual percentage superimposed (solid line). It is not surprising that over long horizons these projections are uninformative. By law, the CBO's projections are based on current law, which is a poor predictor of long-run policy. For example, the projections in the early-mid 1990s did not initially account for the Balanced Budget Act of 1997 and in the early 2000s the projections did not initially account for the U.S. involvement in Iraq and Afghanistan or the Bush tax cuts in 2001. These major policy changes suggest that the debt target switches over time and that households must form a probability distribution over these outcomes to set their expectations of the future debt target and tax policy. Figure 3b plots the distribution of the difference between the CBO's projection in any given year



(a) CBO baseline projections of debt-to-GDP percentages (dashed lines) from Economic and Budget Outlook reports (1986-2010) and Five Year Budget Projection reports (1982-1984) compared to actual debt-to-GDP percentages (solid line).



(b) Distribution of the difference between the CBO's baseline projections of the debt-to-GDP percentage and the actual debt-to-GDP percentage across projection years. The solid line is the median and the dashed lines are the 25/75 percentiles.

Figure 3: Comparison of CBO's baseline projections of the debt-to-GDP percentages with the actual values.

and the actual debt-to-GDP percentage. Over shorter horizons, major changes in policy are less likely to occur and, on average, the CBO's projections are more accurate. The accuracy of the CBO's projections over a 5-year time span suggests that changes in the debt target are infrequent.

The CBO's projections provide a sense for how often the debt target switches, but they do not provide precise estimates. We apply the Gibbs sampler—the tool households use to learn in Case 2—to historical data to set each household's prior transition matrix. Figure 4 shows estimates of the observed transitions and the discretionary tax shocks in the data. The left panels show the average tax rate and debt-to-GDP ratio from 1961-2011. The tax rate is calculated as the share of tax revenue-to-GDP. The upper right panel shows the observations of x_t implied by the data. We apply the Gibbs sampler to this data assuming (4) is the data generating process, $\gamma = 0.30$, and there are three debt target states. We place a dogmatic prior on the standard deviation of discretionary tax shocks equal to the OLS estimate of 0.02. The values of the intercepts are set equal to the $10/50/90$ percentiles of the observations from the data. The sampler determines which intercept best explains the data at any given point in time and yields a sequence of debt target states. The intercepts are shown with a dashed line in the upper right panel. The implied tax shocks are shown in the bottom right panel. The standard deviation of those shocks is 0.013, which is our posterior

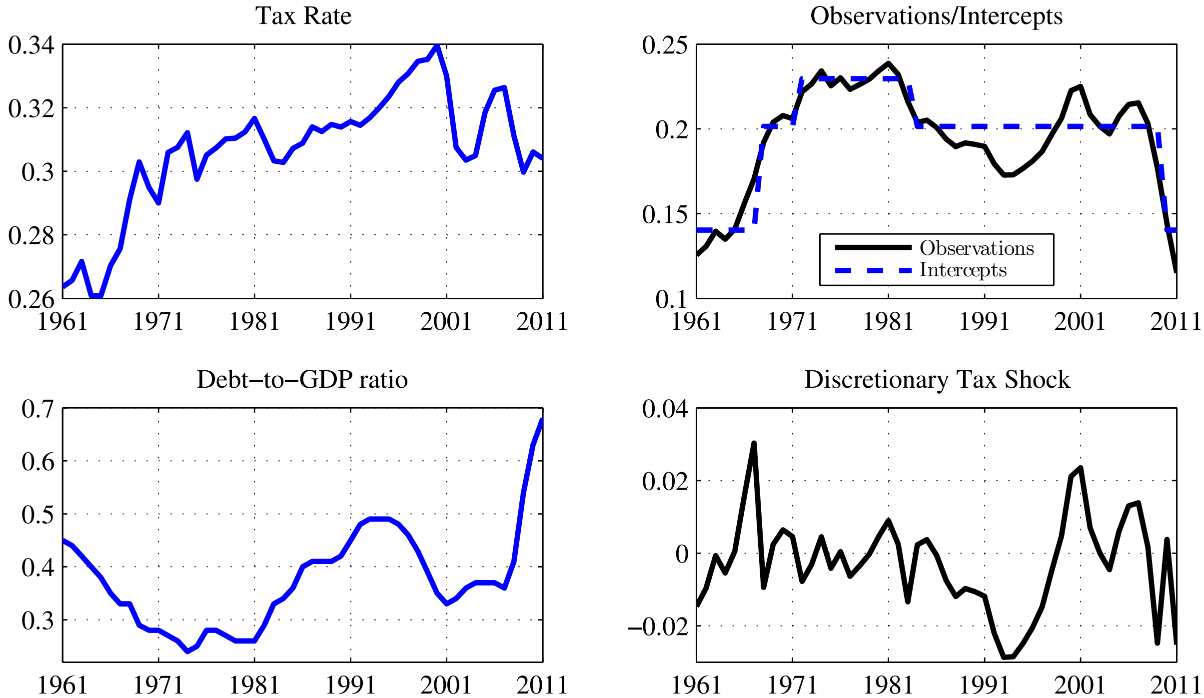


Figure 4: The upper-left-hand panel plots the share of tax revenue-to-GDP from the Bureau of Economic Analysis. The lower left-hand panel plots the debt-to-GDP ratio from the Economic Report of the President. The tax rate and the debt-to-GDP ratio are then used to calculate $x_t = \tau_t - \gamma b_{t-1}/y_{t-1}$, given $\gamma = 0.3$. x_t is plotted with a solid line in the upper-right-hand panel. The dashed line in the upper-right panel represents $\bar{\tau}(s_t) - \gamma\overline{by}(s_t)$, where s_t at each point in time is obtained from applying the Gibbs sampler to the observations (solid line), x_t. The discretionary tax shock in the lower-right-hand panel is solved for from the definition of x_t, $\varepsilon_t = x_t - (\bar{\tau}(s_t) - \gamma\overline{by}(s_t))$.

estimate of σ_ε. The average probabilities and each of their 68% credible intervals are

$$P_{16} = \begin{bmatrix} 0.78 & 0.11 & 0.05 \\ 0.07 & 0.81 & 0.05 \\ 0.07 & 0.12 & 0.66 \end{bmatrix}, \quad \bar{P} = \begin{bmatrix} 0.81 & 0.12 & 0.07 \\ 0.08 & 0.84 & 0.08 \\ 0.10 & 0.18 & 0.72 \end{bmatrix}, \quad P_{84} = \begin{bmatrix} 0.83 & 0.15 & 0.08 \\ 0.10 & 0.87 & 0.11 \\ 0.12 & 0.24 & 0.79 \end{bmatrix}.$$

We define $\bar{P}$ as the prior transition matrix, which is robust to alternative priors for σ_ε. These estimates imply that the debt target changes on average every 5 years, as figure 3 suggests.

4 THE AMPLIFICATION OF TAX SHOCKS

We first plot impulse responses to a two standard deviation negative discretionary tax shock in figure 5 conditional on Case 0. The responses are shown in deviations from steady state and the state is fixed at the mid debt target. The negative shock reduces the tax rate in period 1. A lower tax rate raises the after-tax return on capital and labor, which increases hours worked and investment. That response raises output and consumption. The larger tax base is insufficient to offset the decrease in the tax rate and thus debt rises relative to output. Higher outstanding debt increases the tax rate above its steady state in period 2, which reduces labor hours, investment, and output.

To demonstrate that limited information amplifies the effect of tax shocks, figure 6 plots the same impulse responses as figure 5, but for Case 1. We show the responses to a positive (dashed

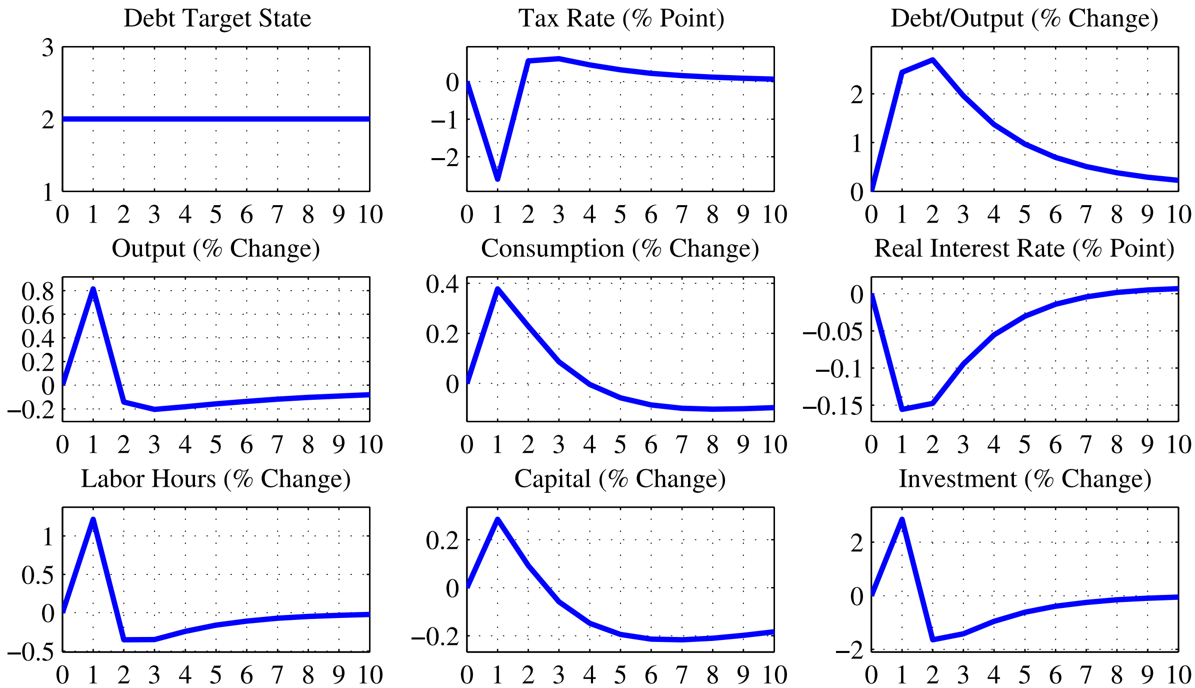


Figure 5: Case 0 impulse responses to a two standard deviation negative discretionary tax shock in percent deviations from the stochastic steady state. The debt target state is fixed at its middle value.

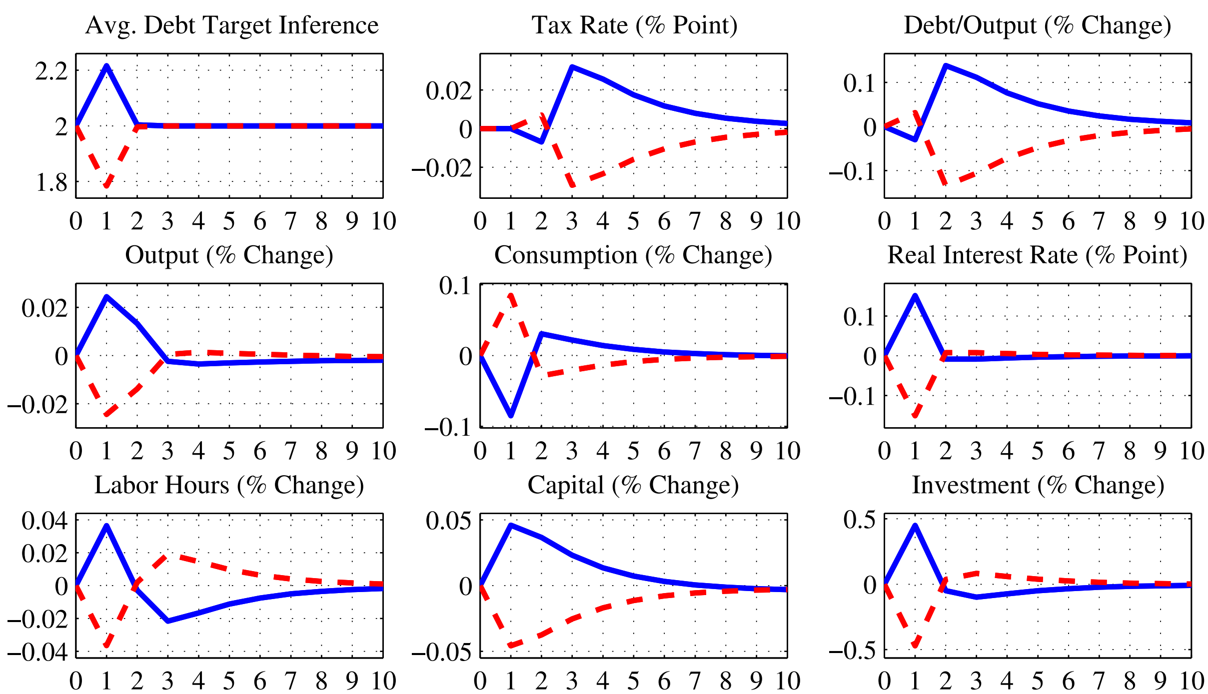


Figure 6: Case 1 impulse responses to a two standard deviation positive (dashed line) and negative (solid line) discretionary tax shock in percent deviations from full information (Case 0). The true state is always the mid debt target.

line) and negative (solid line) shock as percent deviations of Case 1 from Case 0 to capture any asymmetries and isolate the effects of the incorrect inference. The true debt target is fixed at its middle value (75% of output) throughout the simulation, and the tax rate initially corresponds to that target. Consider a scenario where households incorrectly infer that the debt target state rose in period 1 due to a cut in the tax rate (solid line). Since the tax rate is based on the true debt target state and the past debt-to-output ratio, it does not deviate from Case 0 in period 1. However, households increase their labor supply and substitute away from consumption in favor of investment because they expect lower future taxes. The deviation of their expectations from Case 0 is what amplifies the effect of the tax cut and leads to higher output on impact than in Case 0. The real interest rate rises due to the increase consumption growth. With a broader tax base debt falls. In period 2, households realize their inference of the debt target was incorrect and decrease their labor supply and investment in anticipation of higher future taxes. The higher real interest rate in period 1 causes debt to rise in period 2. In period 3, the tax rate increases in response to the higher debt, which further reduces labor and investment. Thus, limited information about the debt target state causes the endogenous variables to differ from the Case 0 paths, even after beliefs are perfectly aligned with the truth. A positive shock has roughly symmetric effects on the economy.

Figure 7 shows the cumulative effects of limited information in Cases 1 and 2. The top panel compares the average inference about the debt target state in Case 1 (solid line) and Case 2 (circle markers) against the true state (dashed line). The average inference is calculated as the realizations of s_t weighted by their conditional probability (i.e., $\sum_{j=1}^{m} j \cdot \Pr[s_t = j|\mathbf{x}^t]$). The two middle panels plot the paths of output in Cases 1 and 2 in percent deviations from Case 0. The bottom panel plots the discretionary tax shock, which obscures the state and affects their inferences.

The paths of output contain sequences of spikes that correspond to the differences between the average inference about the debt target state and the true state.[16] A dark-shaded (light-shaded) region means the inference is higher (lower) than the true debt target by more than one standard deviation of the differences over the entire simulation. The direction is important because incorrect inferences can either increase or decrease households' expectations of the future tax rate. If they infer that the debt target is higher (lower) than the truth, then they expect the tax rate will decrease (increase) to allow debt to rise (fall) toward its target. This is why output in Case 1 is generally higher (lower) than output in Case 0 when their inferences are above (below) the truth.

In Case 1, households rationally learn about the debt target state. They are aware that their inference about the state may be wrong and account for the unknown state by weighting each possible realization by its conditional probability. The deviations of Case 1 from Case 0 are a function of how far their inference is from the truth. Moreover, households are more responsive to discretionary tax shocks than in Case 0. Occasionally, households incorrectly infer that the debt target switched to a higher (lower) target. This means they put positive weight in expectation on the choices they would make conditional on the higher (lower) debt target, which causes their expectation of future tax rates to move in the same direction as the tax shock. Thus, the effects of the tax shocks on output are amplified by the unknown debt target. For example, the bottom panel shows a large negative discretionary tax shock in period 53. This shock causes households to believe the debt target has increased when in fact the middle target is the truth. The path of output in Case 1 rises above Case 0 due to the belief that low short-run tax rates will continue.

[16] Appendix A.1 shows that limited information also generates spikes in the standard deviation of the output forecast error. That is the same measure of uncertainty the SV literature uses, except the source of uncertainty is endogenous.

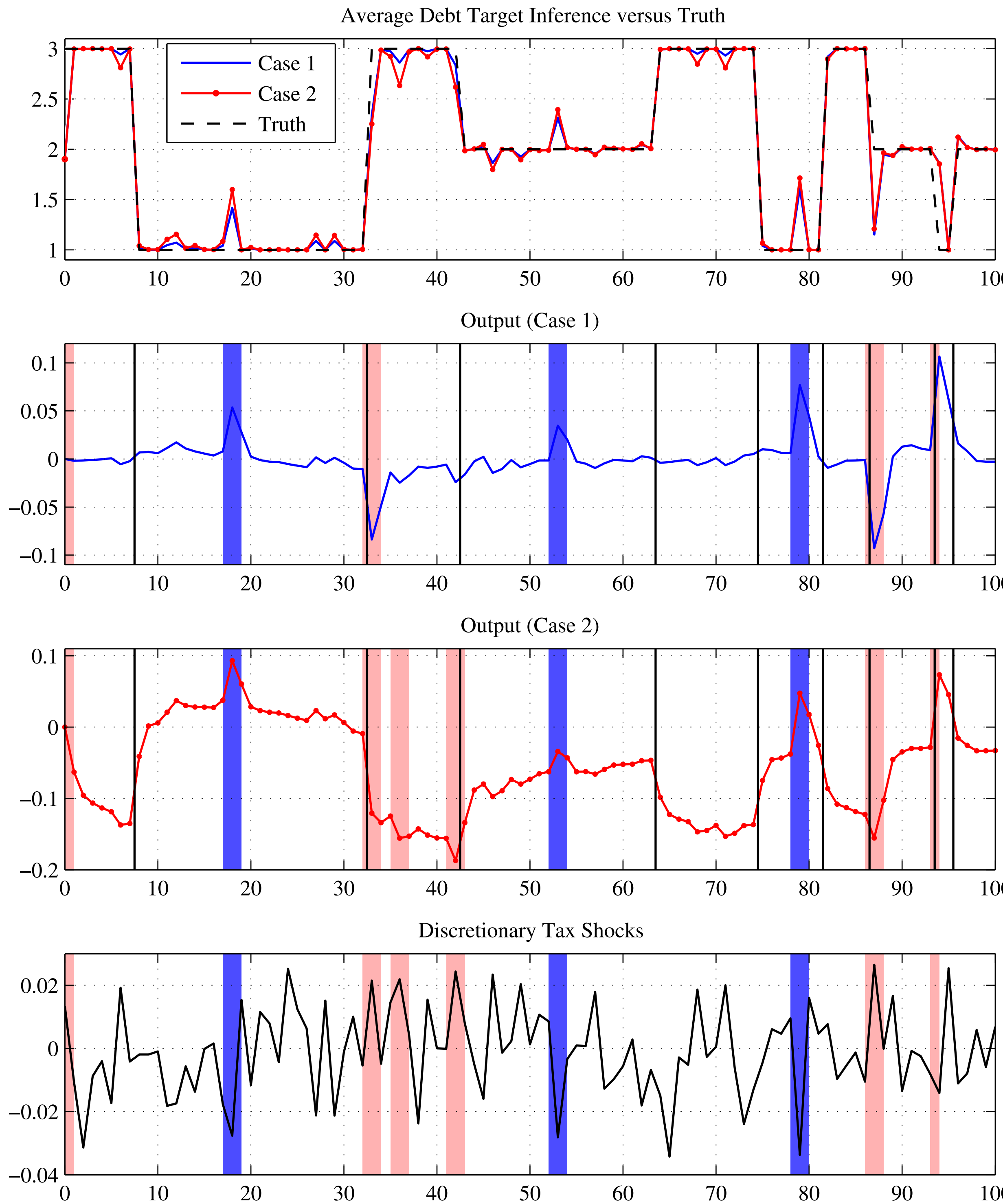


Figure 7: Effect of limited information on households' decisions. Top panel: Household's inference of the state in Case 1 (solid line) and Case 2 (circle markers), calculated as the average of s_t weighted by the conditional probabilities. Middle Panels: Paths of output in Cases 1 and 2 in percent deviations from Case 0 output. Bottom panel: Discretionary tax shocks. A dark-shaded (light-shaded) region means households' inferences about the state are higher (lower) than the true value by one standard deviation. The vertical lines denote periods when the true debt target state changes.

In Case 2, the unknown transition matrix creates more persistent deviations from Case 0, which are interrupted by sequences of spikes when the average inference of the state differs from the truth. Since households know the true transition matrix is time invariant, they use their current estimate of the transition matrix when forming expectations (i.e., they assume it does not change). Initially, observations of different transitional events are rare, which creates bias in the estimates toward those that have been observed. The strong influence of the prior prevents drastic changes to their estimates, but the differences in output between Cases 1 and 2 are still stark. There is also interaction between the unknown state and transition matrix that further amplifies the tax shocks. In general, the inference of the debt target state in Case 2 is further away from the truth than in Case 1 because $\{s_t\}_{t=0}^T$ is sampled sequentially with P, given each household's observations.

In summary, the unknown state causes high frequency deviations from the full information case, while the unknown transition matrix causes lower frequency deviations. Both cases amplify the effects of tax shocks through changes in expected tax rates, which have a cumulative effect on the capital stock. When the debt target is unknown, average output is 0.011% lower (0.012% higher) in states where capital is below (above) its full information value. When the transition matrix is also unknown, average output in the same states is 0.048% lower (0.041% higher).

5 THE WELFARE COSTS OF LIMITED INFORMATION

This section presents the welfare costs in Cases 1 and 2. The results are based on $5{,}000$ simulations of the model. We calculate welfare by thinking of limited information decisions as alternatives to those under full information. In each limited information case, ℓ, we solve for a λ^ℓ that satisfies

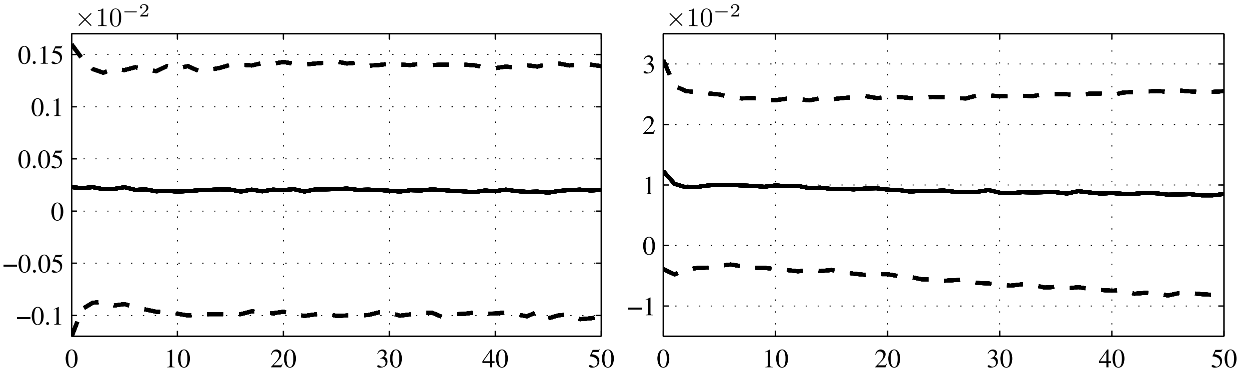
$$\mathbb{E}_t^1 W(c_t(\mathbf{z}_t^1), n_t(\mathbf{z}_t^1)) = \sum_{i=1}^{m} \mathbf{q}_t(i) \mathbb{E}_t^0 W((1-\lambda^1)c_t(\mathbf{z}_t^1|s_t), n_t(\mathbf{z}_t^1|s_t)), \quad \text{(Case 1)}$$

$$\mathbb{E}_t^2 W(c_t(\mathbf{z}_t^2), n_t(\mathbf{z}_t^2)) = \sum_{i=1}^{m} \mathbf{q}_t(i) \sum_{j=1}^{m} \hat{p}_{ij} \mathbb{E}_t^0 W((1-\lambda^2)c_t(\mathbf{z}_t^2|s_t, s_{t+1}), n_t(\mathbf{z}_t^2|s_t, s_{t+1})), \quad \text{(Case 2)}$$

where $W(c_t(\mathbf{z}_t^\ell), n_t(\mathbf{z}_t^\ell)) \equiv \sum_{h=t}^{T-1} \beta^{h-t} u(c_h(\mathbf{z}_h^\ell), n_h(\mathbf{z}_h^\ell))$ for $\ell \in \{1,2\}$ and the indices i and j indicate the realizations of s_t and s_{t+1}, respectively. The decision rules on the right-hand-side are conditional on specific realizations of the state in period t for Case 1 and periods t and $t+1$ for Case 2, while all other realizations of the state are random. The expectations in Case 0 are formed conditionally on those realizations. T is the simulation length, so W is the time-t present value of remaining simulation utility.[17] The left-hand-side is the expected welfare from staying in limited information case ℓ and the right-hand-side is the expected welfare from moving to the full information case from limited information case ℓ. In moving to full information, households weight the probability that they will be revealed the current state by their limited information beliefs. A positive (negative) λ^ℓ represents a welfare cost (benefit) in limited information case ℓ relative to Case 0. We initialize the welfare calculations in period 0 at the stochastic steady state and the calculations in period $t > 0$ at the limited information state vector from period $t-1$.[18]

[17]Since the calculation is based on a simulation, the length of the interval $[t, T]$ must ensure that the present value at t of utility near the end of the simulation is close to zero. We choose $T = t + 501$, noting that $\beta^{500} \approx 3 \times 10^{-9}$.

[18]An alternative welfare measure, which is popular in the heterogeneous agent literature, is *ex-ante* welfare as described by Aiyagari and McGrattan (1998). An *ex-ante* welfare criterion measures the welfare of the agent prior



(a) Case 1 (b) Case 2

Figure 8: The distribution of welfare costs (λ^ℓ) as a percent of Case 0 consumption goods forgone over the remaining simulation. The solid line is the median and the dashed lines are the $16/84$ percentile bands. A positive (negative) number represents a welfare cost (benefit). The values are based on $5{,}000$ Monte Carlo simulations.

Figure 8 compares the distribution of λ^ℓ across Cases 1 and 2 at the median (solid line) and the $16/84$ percentile bands (dashed lines). In Case 1 (figure 8a), households can be compensated for not knowing the current debt target state in the median simulation with around 0.0002% of full information consumption goods. In more than half of the simulations in a given period, limited information about the debt target state causes welfare losses. Moreover, in the tails of the welfare distribution, the welfare costs are larger than the welfare gains. As time evolves, the consumption goods required to compensate households for the limited information in the median simulation does not diminish since their inferences only depend on inferences from the previous period.[19]

How can an unknown debt target lead to welfare gains? In Case 0, households know the current state but not future states. This means households must form expectations over future states to inform their decisions. When the current debt target state is also unknown, households' expectations are potentially misaligned with Case 0, which may lead to welfare gains from overaccumulating capital. To see this more clearly, figure 9 shows households may gain or lose when the average inference about the debt target is misaligned with the truth. For example, if they expect higher taxes due to an average inference that is below the truth, they invest less relative to Case 0. If the debt target winds up higher than expected, welfare is lower than Case 0. With less investment after-tax returns are lower, which reduces future consumption, leisure, and welfare, despite initially higher consumption and leisure. If the debt target winds up lower than expected, the lower level of investment is better aligned with the realization of taxes, and welfare is higher than Case 0.

Figure 8b plots welfare in Case 2, which shows that the unknown transition probabilities have a larger effect than the unknown state. The elements of the *true* transition matrix are set to $p_{ii} = 0.9$, $p_{ij} = 0.05$ for $i \neq j$, and the prior is set to $\bar{P}$ as estimated in section 3. Three facts stand out relative to Case 1: the welfare cost distribution remains fairly symmetric, the compensation is much larger in the tails, and the compensation diminishes as households learn the transition

to the realization of initial assets. However, Li and Sarte (2006) point out that the *ex-ante* welfare criterion ignores transition costs that manifest from policy changes. The welfare changes following a switch in the debt target state are of central importance. Thus, we use a welfare measure that accounts for these costs conditional on the initial realization of assets in limited information case ℓ. Moreover, anticipated utility does not permit *ex-ante* welfare calculations.

[19] Appendix A.2 shows the welfare distributions when the debt targets are set to 40%, 60%, and 80% of output.

matrix. The explanation for both welfare gains and losses is similar to Case 1, but the source of the welfare consequences is different. Given a prior on the transition matrix consistent with past data, households initially expect the low debt target will be realized more often. However, their initial observations can bias their estimates, which affects their decisions relative to Case 0. More specifically, if households observe a particular sequence of states that favors one target more than the others, then they will over-estimate the probability of switching to and remaining at that target, which misaligns their expectations with the truth. As an example, suppose a simulation remains in the high debt target state longer than the true expected duration (10 years). This causes households to infer that a higher debt target (a lower tax rate) is more likely and to increase their investment relative to Case 0. If the debt target remains high in the future, households benefit since their after-tax returns will be higher than in Case 0. However, if the debt target falls in the future, households lose, since they face higher taxes. Some of the draws of the state will be shorter and some will be longer than the true expected duration, which means both welfare gains and losses are possible.

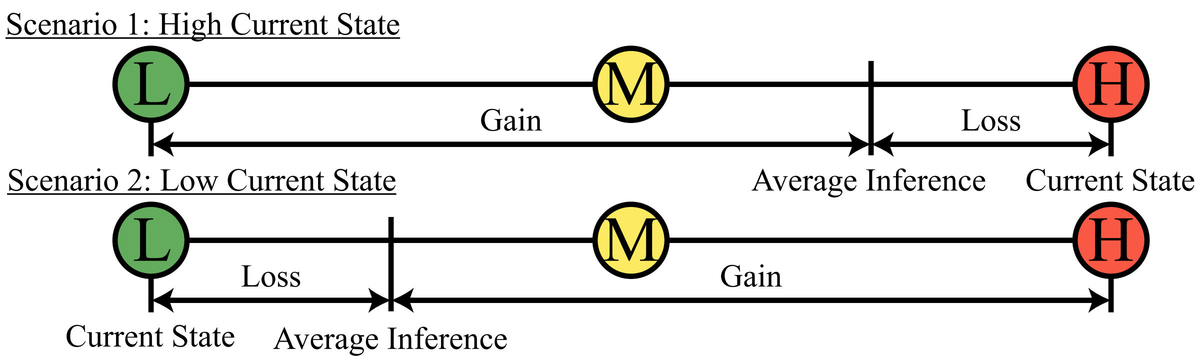


Figure 9: An illustration of welfare gains and losses depending on the future realization of policy.

Household requires more consumption goods to compensate them for the unknown transition matrix than the unknown state for two reasons. First, the transition matrix is more complicated to learn. It is defined by six parameters and requires a large sample of observations to obtain an accurate estimate. Second, households treat their estimate of the transition matrix as the truth in expectation. Their estimate changes across time, but in expectation they believe it is the truth and time invariant. Thus, their suboptimal decisions persist and the welfare consequences are more severe than in Case 1. As households gather more observations, on average their estimates approach the truth, which reduces the median of the welfare costs. Initially, households require 0.012% of full information consumption goods to compensate them for the unknown transition matrix in the median simulation, which declines slowly since the transition matrix is hard to learn.

If we included a productivity shock in our model, it should not affect the welfare results since the endogenous part of the tax rule, $\tau_t - \gamma b_{t-1}/y_{t-1}$, can be expressed as a function of two hidden exogenous components, the unknown debt target state and the tax shock. If the productivity shock is observed, then households only need to disentangle those two components. The Gibbs sampler used to estimate the transition matrix also requires knowledge of the tax rule in the same way. Thus, we speculate that the welfare results in Case 2 would be unaffected by an observed productivity shock. However, *unobserved* productivity shocks would affect the welfare results since the tax rule, which is a function of output, would contain more noise that further obscures the unknown debt target state. Households would need to determine how much of a change in the tax rate is due to productivity shocks in addition to the two fiscal shocks, which would most likely increase the consequences of an unknown debt target much like an increase in the variance of the tax shock.

6 Recent Fiscal Policy and the Costs of Uncertainty

The previous section makes clear that depending on how households' expectations are aligned with the realization of future states, limited information can result in welfare gains or losses. Those results may seem counterintuitive, given the popular belief that fiscal uncertainty leads to welfare losses. This section uses our model to interpret the uncertainty surrounding the passage of the ARRA in 2009 and the scheduled sunset of the Bush tax cuts in 2011. We show in the following experiments that limited information is likely to reduce output, investment, and welfare.

6.1 American Recovery and Reinvestment Act The ARRA was an economic stimulus package passed in February 2009 to combat the effects of the Great Recession. The legislation included a combination of tax incentives ($288 billion), aid to state and local governments ($144 billion, mostly for education and Medicaid), and federal spending ($357 billion, mostly for infrastructure, education, and aid to low-income workers and the unemployed) totaling $789 billion.

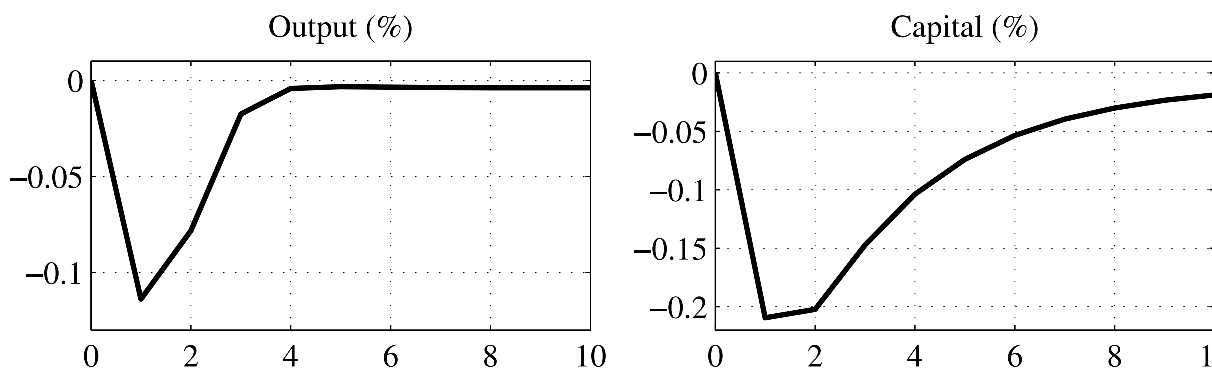


Figure 10: Consequences of limited information in Case 1 when there a simultaneous increase in the debt target state from low to mid and a discretionary tax shock. The values are given in percent deviations from Case 0.

The ARRA led to a sharp increase in the debt-to-GDP ratio, which was partly due to the tax cuts. Using our model, we can quantify the effect of the uncertainty that households faced about whether debt would rise to a new long-run level or whether future policy would adjust to maintain the current debt level. The uncertainty stems from the fact that there were strong calls for fiscal consolidation, particularly from newly elected members of the Tea Party, while the economy was in a recession, which may signal to households that Congress is willing to allow the debt target to rise to stimulate the economy. In Case 1, households may mistake a tax shock for a change in the debt target. Moreover, a tax shock can obscure a switch in the debt target if it happens simultaneously. Figure 10 shows the effects on output and the capital stock in Case 1 when there is a simultaneous (and permanent) increase in the debt target state from low to mid and a two standard deviation discretionary tax shock in period 1. To isolate the effect of the limited information, the paths are shown as percent deviations from Case 0. Given the simultaneous change in policies, the net effect is a tax cut, which is consistent with the ARRA. On impact, the change in policies lead households to believe the current debt target is at its middle value with a probability of only 1.7%, while they place a probability of 98.3% on the low debt target. Thus, they expect higher future taxes relative to Case 0. That expectation leads to a decrease in both hours worked and investment, which reduces output below its full information value by over -0.1%. These results show that limited information about the future direction of policy reduced the stimulative effect of ARRA.

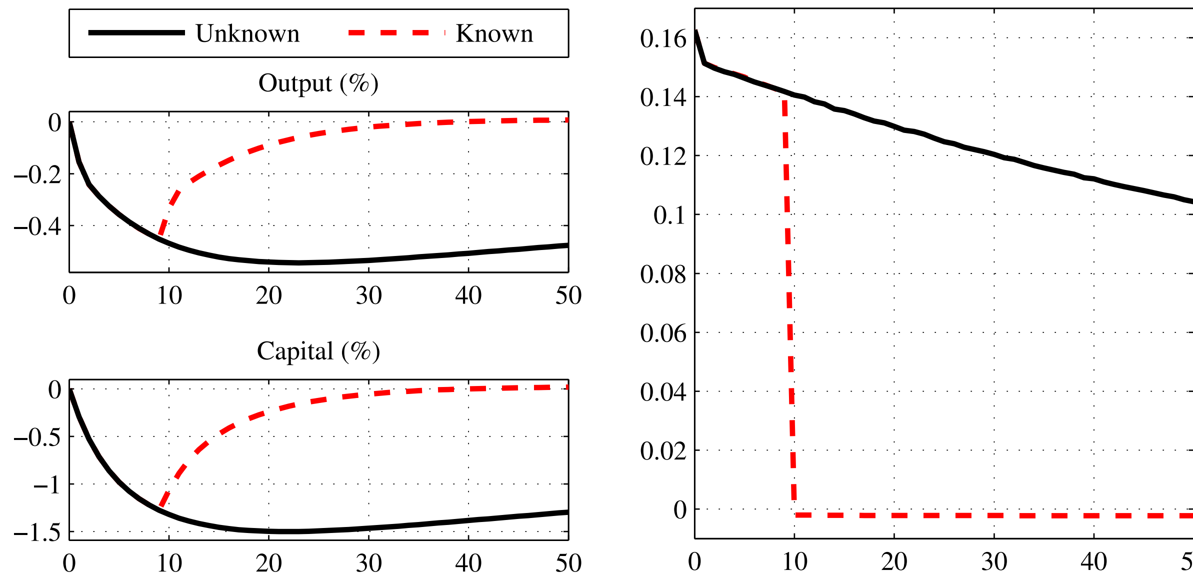
6.2 BUSH TAX CUTS Next, we review the legislation responsible for the Bush tax cuts, which were passed in 2001 and later extended. The Economic Growth and Tax Relief Reconciliation Act of 2001 was initially passed to return budget surpluses to taxpayers but was later promoted to counteract the recession. In addition to a one-time tax rebate (\$$300$ for individuals, \$$600$ for married couples), it replaced the existing five income tax rates (15%, 28%, 31%, 36%, and 39.6%) with six lower rates (10%, 15%, 25%, 28%, 33%, and 35%). Tax rates of 28% or higher were immediately reduced by 0.5% and further reductions were scheduled to occur in 2002, 2004, and 2006. These rates were then scheduled to revert back to their original values (i.e., sunset) in 2011.

The Jobs and Growth Tax Relief Reconciliation Act of 2003 advanced the final implementation of the cuts from 2006 to 2003 while maintaining the scheduled sunset and also reduced the top (bottom) tax rate on capital gains to 15% (5%). The capital gains rate reductions were originally scheduled to sunset in 2008 but later extended through 2010 by the Tax Increase Prevention and Reconciliation Act of 2005. Together the tax acts of 2001 and 2003 are commonly referred to as the Bush tax cuts. The months leading up to the expiration of the Bush tax cuts were filled with uncertainty about whether a politically divided Congress would reach an agreement to extend the tax cuts. In late December 2010, just before the tax cuts were set to expire, Congress passed the Tax Relief, Unemployment Insurance Reauthorization, and Job Creation Act, which extended the Bush tax cuts for an additional 2 years. That legislation lead to more speculation about whether the tax cuts would be permanently extended or allowed to expire. In a last minute deal, the American Taxpayer Relief Act of 2012 was passed on January 1st, 2013 to permanently extend the Bush tax cuts for individuals with taxable income of \$$400{,}000$ (\$$450{,}000$ if married) per year or less. For those with higher incomes, the top income tax rate returned to 36.9% and the top capital gains tax rate increased to 20%, which was consistent with the scheduled sunset of the 2001 Bush tax cuts.

Limited information about whether the Bush tax cuts would sunset likely depressed output and led to welfare losses. To understand why, suppose Congress announces that they will reduce deficits with an increase in taxes. Households believe the announcement is credible and put probability on higher short-run taxes. However, when it comes time to increase taxes, Congress reneges on its promise and keeps taxes low. Households are surprised to see taxes remain low and invested less than they would have if they knew taxes would stay low. That mistake leads to persistently lower output, which the economy slowly recovers from as households revise their expectations.

To quantify the effects of this policy, we consider an experiment where the true debt target state is always high (i.e., there is no uncertainty about future debt target states in Case 0 since the high debt target state is perfectly absorbing), but households face limited information about the current state and how it changes over time. The solid lines in figure 11 plot the Case 2 paths of output and capital in percent deviations from Case 0 and the welfare costs. The simulation is initialized with the estimated transition matrix, $\hat{P}_0$, equal to $\bar{P}$ and conditional probabilities, $\mathbf{q}_0$, equal to the ergodic distribution of $\bar{P}$. Thus, our results are conditional on the U.S. data embodied in $\bar{P}$.

Learning results in lower capital, consumption, and leisure. Households expect the low debt target to occur more often so that realizations of the tax rate are strictly below expectations. Each period households are surprised by low taxes. These surprises decline over time as households' es-



(a) Simulated paths

(b) Welfare costs

Figure 11: Consequences of limited information in Case 2 when the true debt target is always high (solid line) and the benefit of knowing that target (dashed line). The values are given in percent deviations from Case 0.

timates of the transition matrix approach the truth. Initially, the surprises are large and households invest much less than in Case 0 due to higher expected taxes. Their belief of the transition matrix adjusts rapidly, which leads to higher investment and a build-up of the capital stock. Thus, capital accumulates to the level in Case 0, at the expense of lower consumption, leisure, and welfare. This example suggests that the recent debate surrounding the Bush tax cuts caused households to underinvest in capital and slowed the recovery from the Great Recession.

The dashed lines in figure 11 show the effect of the fiscal authority deciding to reveal the high debt target to guide households' expectations after a prolonged period of limited information. The announcement in period 10 expedites the recovery to full information output. The experiment is the same as the one described above, except beginning in period 10 households' beliefs coincide with the true transition matrix (i.e., they know the high debt target is absorbing). Once their expectations match actual fiscal policy dynamics, output nearly makes a full recovery within 10 years. Therefore, the fiscal authority is able to effectively counteract the negative consequences of households' limited information from the first 10 years of the simulation. Once again, these results illustrate the potential consequences of households having limited information about future policy.

7 CONCLUSION

Our main contribution is to model the uncertainty surrounding fiscal policy by account for the fact that households have limited information about the current debt target and how it evolves over time. They are uncertain about whether short-run changes in tax policy imply a change in the long-run level of debt or whether future policy will return debt to its current long-run level. Households rationally learn about the debt target and use anticipated utility to form expectations conditional on their estimates of the transition matrix. We analyze the effects of that limited information on households' decisions, quantify the welfare consequences, and examine consequences of the uncertainty surrounding recent policies.

Discretionary tax shocks cause households to incorrectly infer a change in the debt target. That inference causes households to form expectations about future taxes under limited information differently than they would under full information, which affects their consumption/saving decisions. Those alternative decisions amplify the effects of tax shocks and are more likely to cause welfare losses than welfare gains. Moreover, an unknown debt target likely reduced the stimulative effect of the ARRA and uncertainty about the sunset provision in the Bush tax cuts may have slowed the recovery and led to welfare losses. These results suggest that the economy would benefit from a committee that communicates fiscal targets to the public and is insulated from the political process.

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A ADDITIONAL RESULTS

A.1 ENDOGENOUS UNCERTAINTY This section draws a contrast between the uncertainty in the stochastic volatility (SV) literature and type of uncertainty we consider. The SV literature studies the effects of exogenous changes in uncertainty. As an example, suppose a model includes an exogenous random variable x, such as government spending, that evolves according to $x_t = \rho_x x_{t-1} + \sigma_t \varepsilon_t$, where $0 \leq \rho_x < 1$ and ε is white noise. SV is introduced into the model by assuming the standard deviation of the shock is time-varying and follows an exogenous process specified by the modeler. The uncertainty is measured by the expected volatility of the forecast error, which is given by

$$\sqrt{E_t[FE^2_{x,t+1}]} = \sqrt{E_t[(x_{t+1} - E_t x_{t+1})^2]} = \sqrt{E_t[(x_{t+1} - \rho_x x_t)^2]} = \sqrt{E_t \sigma^2_{t+1}}.$$

Thus, time-varying changes in the shock variance will cause periods of high and low uncertainty.

Limited information generates time-varying uncertainty endogenously. Figure 12 shows the standard deviation (SD) of the one-period ahead forecast error (FE) for output, given by,

$$\sigma^\ell_y \equiv \sqrt{E_t[(FE^\ell_{t+1} - E_t[FE^\ell_{t+1}])^2]},$$

where $FE^\ell_{t+1} = y^\ell_{t+1} - E_t y^\ell_{t+1}$. This statistic quantifies the uncertainty surrounding output. As in figure 7, the shaded regions indicate periods of incorrect inference about the debt target state, and the solid vertical lines represent switches in the debt target. The SD of the output FE is plotted relative to the uncertainty in Case 0 (i.e., $\sigma^\ell_y - \sigma^0_y$, $\ell \in \{1, 2\}$). An unknown debt target and transition matrix generates the periodic bursts of uncertainty seen in the data. The two sources of high uncertainty are large discretionary tax shocks and changes in the debt target. The uncertainty occurs in short bursts around those events and then returns to a value that is consistent with the level of uncertainty under full information. Typically, uncertainty is higher in Case 2 than Case 1.

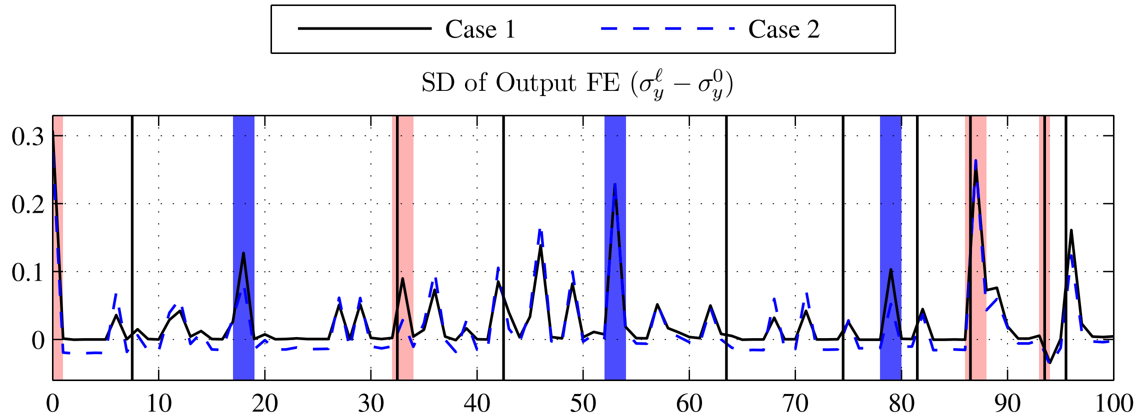

Figure 12: The difference between the limited and full information standard deviation of the output forecast error. A dark-shaded (light-shaded) region means the belief is higher (lower) than the true value by one standard deviation. The vertical lines denote periods when the true debt target state changed.

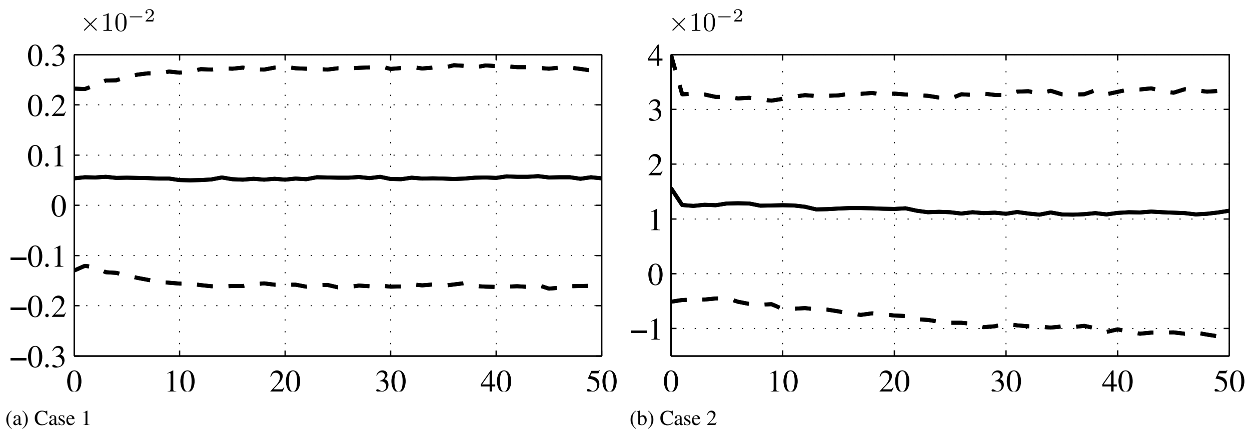

Figure 13: Welfare results when the debt targets are set to 40%, 60%, and 80% of output. The distributions of the welfare costs are shown as a percent of Case 0 consumption goods forgone over the remaining simulation. The solid line is the median and the dashed lines are the $16/84$ percentile bands. A positive (negative) number represents a welfare cost (benefit). The values are based on $5{,}000$ Monte Carlo simulations.

A.2 ROBUSTNESS OF ALTERNATIVE DEBT TARGET VALUES Figure 13 plots the welfare distributions when the possible debt targets are reduced to 40%, 60%, and 80% of output (σ_ε is unchanged), which allows for the possibility that U.S. debt-to-GDP returns to its historical average of 40%. Relative to the distributions shown in figure 8, the median welfare cost in Case 1 is about twice as large, and the losses in the tail are almost twice the size. In Case 2, there is a slight increase in the median and tail welfare costs. In both cases, the welfare benefits in the tail of the distributions decrease. The ability of households to infer the debt target state crucially depends on the ratio of the variance of the tax shock and the distance between the debt targets. For a given set of debt targets, an increase in the variance of the tax shock makes it more difficult for households to infer the debt target state and estimate the transition matrix, which leads to greater welfare losses. For a given variance of the tax shock, an increase in the distance between the debt targets (in this case from 15% to 20% of output) makes it easier for households to infer the debt target state. However, the consequences of incorrectly inferring the state increase since there is a greater difference between expected and realized future tax rates when the debt targets are further apart. In this example, that affect dominates any improvements in welfare from a smaller ratio of the variance of the tax shock to the distance between the debt targets.

B NUMERICAL METHODS

B.1 SOLUTION METHOD Policy function iteration approximates the vector of decision rules, Φ, as a function of the state vector, $\mathbf{z}^\ell$. The time-invariant decision rules for the model are

$$\underbrace{\Phi(\mathbf{z}_t^\ell)}_{\text{True RE Solution}} \approx \underbrace{\hat{\Phi}(\mathbf{z}_t^\ell).}_{\text{Approximating Function}}$$

We iterate on $\Phi = (n, r)$, the labor and interest rate policy functions. Each continuous state variable in $\mathbf{z}^\ell$ is discretized into N_d points, where $d \in \{1, \dots, D\}$ and D is the dimension of the state space. The discretized state space is represented by a set of unique D-dimensional coordinates (nodes). For the continuous endogenous state variables, we set the lower (upper) bound to 5% below (above) the minimum (maximum) fixed-regime deterministic steady state value [e.g., the steady state tax rate is lowest (highest) in the low (high) debt target state and so its lower (upper) bound is $0.95\bar{\tau}(s = 1)$ $(1.05\bar{\tau}(s = 3))$]. We specify $(5, 7, 7)$ grid points for the (k, rb, τ) state variables. In Case 0, $s \in \{1, 2, 3\}$ has three grid points. In Cases 1 and 2, $\mathbf{q}$ is mapped into $\boldsymbol{\xi} \in \mathbb{R}^2$ according to section B.2. $\boldsymbol{\xi}$ is then discretized into 15 grid points. We set $G = 11$ Gauss-Hermite nodes for the discretionary tax shock.

The following outline summarizes the fixed-point policy function algorithm we employ. Let $i \in \{1, \dots, I\}$ index the iterations of the algorithm and $j \in \{0, \dots, \Pi_{d=1}^D N_d\}$ index the nodes.

1. Obtain the initial labor and interest rate policy functions, $\hat{n}_0$ and $\hat{r}_0$, on each node, from the fixed-regime log-linear model. We use `gensys.m` to obtain these conjectures.
2. For a given limited information set, ℓ, implement the following steps for each i:
   (a) On each node, solve for $\{y_t, w_t, r_t^k, c_t, i_t, k_t, b_t\}$ given $\mathbf{z}_t^\ell(j)$, $\hat{n}_{i-1}(\mathbf{z}_t^\ell(j))$ and $\hat{r}_{i-1}(\mathbf{z}_t^\ell(j))$. Solve for each potential realization of $\tau_{t+1} = \bar{\tau}(s_{t+1}) + \gamma(b_t/y_t - \overline{by}(s_{t+1})) + \varepsilon_{t+1}$, given $\{\varepsilon_{t+1}^g\}_{g=1}^G$ and $s_{t+1} \in \{1, 2, 3\}$.
   (b) If $\ell \in \{1, 2\}$, then update $\boldsymbol{\xi}_t$ given $x_{t+1} = \bar{\tau}(s_{t+1}) - \gamma\overline{by}(s_{t+1}) + \varepsilon_{t+1}$ for all combinations of $\{\varepsilon_{t+1}^g\}_{g=1}^G$ and $s_{t+1} \in \{1, 2, 3\}$. To do this, first map $\boldsymbol{\xi}_t$ into $\mathbf{q}_t$ (see section B.2), then update $\mathbf{q}_t$ with each potential observation x_{t+1} (see section C.1). Finally, map $\mathbf{q}_{t+1}$ back into $\boldsymbol{\xi}_{t+1}$.
   (c) Using a monomial basis, X, approximate n_{t+1} at each node in $\mathbf{z}_{t+1}$, which is comprised of $(k_t, r_t b_t, \tau_{t+1})$ and $s_{t+1} \in \{1, 2, 3\}$ (Case 0) or $\boldsymbol{\xi}_{t+1}$ (Cases 1 and 2). The set of monomials correspond to a second-order Taylor approximation of the labor policy function. New estimates of the coefficients are obtained every iteration using the least squares estimator, $\hat{\eta}_{i-1} = (X_t' X_t)^{-1} X_t' \hat{n}_{i-1}$. The updated policy function is $n_{t+1} = X_{t+1}\hat{\eta}_{i-1}$.
   (d) Integrate across $\{\varepsilon_{t+1}^g\}_{g=1}^G$ and $s_{t+1} \in \{1, 2, 3\}$ (Case 0) or $\boldsymbol{\xi}_{t+1}$ (cases 1 and 2) according to (5) or (6). Each of the G values ε_{t+1}^g are Gauss-Hermite quadrature nodes.

(e) Solve for $\hat{n}_i$ and $\hat{r}_i$ using the first order conditions of the equilibrium system

$$\hat{n}_i = \lambda \left[\frac{(1-\alpha)a(1-\tau_t)k_{t-1}^{\alpha}\mathbb{E}_t\ell[\beta(c_t/c_{t+1})(1-\tau_{t+1})(r_t^k+1-\delta)]}{\chi} \right]^{1/(\alpha+\eta)} + (1-\lambda)\hat{n}_{i-1}$$
$$\hat{r}_i = \lambda \mathbb{E}_t^{\ell}[\beta(c_t/c_{t+1})]^{-1} + (1-\lambda)\hat{r}_{i-1},$$

where $0 < \lambda < 1$ is the update weight that stabilizes the algorithm. We set $\lambda = 0.80$.

3. Define $\mathrm{maxdist}_i \equiv \max\{|\hat{n}_i - \hat{n}_{i-1}|, |\hat{r}_i - \hat{r}_{i-1}|\}$. Repeat the steps in item 2 until $\mathrm{maxdist}_i < 10^{-8}$ for all j.

B.2 DISCRETIZATION METHOD (3-STATE MARKOV CHAIN) Several Markov switching models contain only two states. If it is a hidden Markov model, then a single probability, call it q_t, describes households' beliefs about the state in period t. Households believe with probability q_t that they are in state 1 and probability $1 - q_t$ that they are in state 2. Thus, if the state is unknown at t and there are only two realizations, then discretizing q_t on the interval $[0, 1]$ in the state-space is straightforward. Policy function iteration is used to obtain an approximate solution to the hidden Markov model by including q in the state-space with an appropriate rule to update q to form expectations. However, if there are m states in the Markov chain, then the restriction $\sum_i \mathbf{q}_t(i) = 1$, for $i \in \{1, \ldots, m\}$ where $\mathbf{q}_t(i) = \Pr(s_t = i|\mathbf{x}^t)$, imposes a constraint on the discretized state space. For example, if each $\mathbf{q}_t(i)$ is independently discretized on the interval $[0, 1]$, then the algorithm will attempt to solve the model in regions of the state space that violate the restriction on the probabilities. Thus, we need an alternative way to discretize the state space.

Suppose that $m = 3$, as in our model. The restriction on the probabilities in period t is $\mathbf{q}_t(1) + \mathbf{q}_t(2) + \mathbf{q}_t(3) = 1$. Note that given two of the three probabilities, the third is implied. Geometrically, the restriction is a formula for a plane. The additional restrictions that $0 \leq \mathbf{q}_t(i) \leq 1$ make it an equilateral triangle with vertices at the unit vectors $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$. Therefore, there exists a one-to-one mapping (bijection), $g : \mathbb{R}^3 \to \mathbb{R}^2$, that satisfies the above restrictions (i.e., one that defines the triangle in a 2-dimensional space). If $g(\mathbf{q}_t) = \boldsymbol{\xi}_t$, with $\mathbf{q}_t$ and $\boldsymbol{\xi}_t$ as row vectors, then the mapping g is a projection,

$$g(\mathbf{q}_t) = (\mathbf{q}_t - \mathbf{o})\mathbf{B} = \boldsymbol{\xi}_t,$$

where $\mathbf{o}$ defines the origin of the projection in $\mathbb{R}^3$, $\mathbf{B}$ is a orthonormal basis of the original plane, and $\sum_i \mathbf{q}_t(i) = 1$. To derive $\mathbf{B}$, note that $\tilde{\mathbf{b}}_1 = (0, 1, -1)$ and $\tilde{\mathbf{b}}_2 = (1, 0, -1)$ form a basis for the original plane. We use the Gram-Schmidt process to obtain orthogonal vectors $\mathbf{b}_1$ and $\mathbf{b}_2$, where

$$\mathbf{b}_1 = \tilde{\mathbf{b}}_1 = (0, 1, -1), \qquad \mathbf{b}_2 = \tilde{\mathbf{b}}_2 - \mathrm{proj}_{\mathbf{b}_1}\left(\tilde{\mathbf{b}}_2\right) = (1, -1/2, -1/2),$$

so that $\mathbf{B} \equiv [\mathbf{b}_1^T/||\mathbf{b}_1||, \mathbf{b}_2^T/||\mathbf{b}_2||]$ is an orthonormal basis. To see the mapping clearly, define $\mathbf{o} \equiv (1, 0, 0)$ and write the mapping as a system of two linear equations,

$$\xi_t(1) = (\mathbf{q}_t(1) - 1)b_{11} + \mathbf{q}_t(2)b_{21} + \mathbf{q}_t(3)b_{31}$$
$$\xi_t(2) = (\mathbf{q}_t(1) - 1)b_{12} + \mathbf{q}_t(2)b_{22} + \mathbf{q}_t(3)b_{32}.$$

Imposing $\sum_i \mathbf{q}_t(i) = 1$ and simplifying the system implies

$$\xi_t(1) = \mathbf{q}_t(2)(b_{21} - b_{11}) + \mathbf{q}_t(3)(b_{31} - b_{11})$$
$$\xi_t(2) = \mathbf{q}_t(2)(b_{22} - b_{12}) + \mathbf{q}_t(3)(b_{32} - b_{12}).$$

Thus, given an appropriate choice of $\mathbf{B}$, there is a one-to-one mapping between $\boldsymbol{\xi}_t$ and $\mathbf{q}_t$.

To implement the discretization, $\mathbf{q}_t(2)$ and $\mathbf{q}_t(3)$ are first discretized on the interval $[0,1]$, with $\mathbf{q}_t(1) = 1 - \mathbf{q}_t(2) - \mathbf{q}_t(3)$. Permutations that violate $\mathbf{q}_t(1) \geq 0$ are discarded. The remaining coordinates are mapped into $\boldsymbol{\xi}_t \in \mathbb{R}^2$, which is included in the state space and has the practical benefit of reducing the state space by one dimension. The policy functions are approximated with ordinary least squares on a complete monomial basis of order 2 to avoid complications with ordering the discretized coordinates in a meaningful way.

C Statistical Methods

C.1 Hamilton Filter In Cases 1 and 2, households observe the tax rate and lagged debt-to-output ratio but not the decomposition of the observation between the state-dependent parameters and the discretionary shock. This means households do not know the current debt target state, s. With each additional observation, households use a Hamilton (1989) filter to update their inference about the probability distribution of s. This section outlines the filter, which is an application of Bayes' theorem. For additional information on statistical inference in Markov-switching models see Hamilton (1994) and Frühwirth-Schnatter (2006).

The filter takes as input a vector of conditional probabilities, which are in the state space of each household's optimization problem, given by,

$$\Pr[s_{t-1} = i|\mathbf{x}^{t-1}],$$

where $i \in \{1,2,3\}$ and $\mathbf{x}^{t-1} \equiv \{x_0, \ldots, x_{t-1}\}$, and updates them with the most recent observation, $x_t \equiv \tau_t - \gamma b_{t-1}/y_{t-1}$. The filter outputs the updated conditional probabilities, given by,

$$\Pr[s_t = j|\mathbf{x}^t],$$

where $j \in \{1,2,3\}$, and the conditional likelihood of x_t, $f(x_t|\mathbf{x}^{t-1})$. To apply the filter, note that the probability density of x_t is normally distributed so that

$$f(x_t|s_t = j, s_{t-1} = i, \mathbf{x}^{t-1}) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} \exp\left(-\frac{\varepsilon_t^2}{2\sigma_\varepsilon^2}\right),$$

where $\varepsilon_t = x_t - (\bar{\tau}(s_t) - \gamma\overline{by}(s_t))$. The filter's output is obtained with the following sequence of calculations for all i, j:

1. Calculate the joint probability of $(s_t = j, s_{t-1} = i)$ conditional on $\mathbf{x}^{t-1}$,

$$\Pr[s_t = j, s_{t-1} = i|\mathbf{x}^{t-1}] = \Pr[s_t = j|s_{t-1} = i]\Pr[s_{t-1} = i|\mathbf{x}^{t-1}].$$

2. Calculate the joint conditional density-distribution of x_t and $(s_t = j, s_{t-1} = i)$,

$$f(x_t, s_t = j, s_{t-1} = i|\mathbf{x}^{t-1}) = f(x_t|s_t = j, s_{t-1} = i, \mathbf{x}^{t-1})\Pr[s_t = j, s_{t-1} = i|\mathbf{x}^{t-1}].$$

3. Calculate the likelihood of x_t conditional on $\mathbf{x}^{t-1}$,

$$f(x_t|\mathbf{x}^{t-1}) = \sum_{j=1}^{m}\sum_{i=1}^{m} f(x_t, s_t = j, s_{t-1} = i|\mathbf{x}^{t-1}).$$

4. Calculate the joint probabilities of $(s_t = j, s_{t-1} = i)$ conditional on $\mathbf{x}^t$,

$$\Pr[s_t = j, s_{t-1} = i|\mathbf{x}^t] = \frac{f(x_t, s_t = j, s_{t-1} = i|\mathbf{x}^{t-1})}{f(x_t|\mathbf{x}^{t-1})}.$$

5. Calculate the output by summing the joint probabilities over the realizations s_{t-1},

$$\Pr[s_t = j|\mathbf{x}^t] = \sum_{i=1}^{m} \Pr[s_t = j, s_{t-1} = i|\mathbf{x}^t].$$

Each household's expectations contain future policy variables, which are interpolated at the updated probability distribution for s_t.

C.2 GIBBS SAMPLER In Case 2, when both the current debt target state and transition probabilities are hidden, households use a Gibbs sampler to estimate the transition probabilities (i.e., they construct a Markov chain such that the limiting distribution of the chain is the joint distribution of interest). Household still use the Hamilton filter to infer the current, end-of-sample, state. This is because the Gibbs sampler efficiently relies on the lagged and future states to draw a sequence of states each iteration, and so it performs relatively poorly at the end-of-sample where the future is unobserved. Furthermore, it cannot replace the Hamilton filter in the optimization problem since it requires the entire sample to estimate the probability distribution of s, which is computationally prohibitive.

Albert and Chib (1993) outline a Gibbs sampler to estimate an entire autoregressive 2-state hidden Markov model, but since we assume households understand certain aspects of fiscal policy—the speed of expansion/consolidation, the long-run debt targets and supporting tax rates, and the volatility of *i.i.d.* discretionary tax shocks—the following Gibbs sampler only needs to iterate on the sequence of states and the transition probabilities. We chose a Gibbs sampler rather than maximum likelihood estimation of the Hamilton filter since imposing a prior distribution on the transition matrix yields reasonable estimates with short samples (i.e., it keeps the transition probabilities bounded away from from 0 and 1) and it does not require a nonlinear solver.

In period T, households sample a sequence of states, $\mathbf{s}^T$, conditional on the 3-state transition matrix, P, and the observations, $\mathbf{x}^T$, sequentially. The following steps outline the sampler:

1. Initialize the Gibbs chain by sampling a sequence of states, $\mathbf{s}^T = \{s_1, \ldots, s_T\}$, from the prior transition matrix, P, using a uniform random number generator.
2. For $t \in \{1, \ldots, T\}$ and $j \in \{1, 2, 3\}$, sample s_t conditional on $\mathbf{x}^T$ and the neighboring states from the previous draw using the following rules:

   - If $t = 1$, then $f(s_1|\mathbf{x}^T, \mathbf{s}_{-1}) \propto \Pi_j(P) p_{jk} f(x_1|s_1)$, where $s_2 = k$.
   - If $1 < t < T$, then $f(s_t|\mathbf{x}^T, \mathbf{s}_{-t}) \propto p_{ij} p_{jk} f(x_t|s_t)$, where $s_{t-1} = i$ and $s_{t+1} = k$.
   - If $t = T$, then $f(s_T|\mathbf{x}^T, \mathbf{s}_{-T}) \propto \Pi_j(P) p_{ij} f(x_T|s_T)$, where $s_{T-1} = i$.

   s_t and x_t are particular realizations at time t, $\Pi_j(P)$ is the jth element of the stationary distribution of P, p_{ij} is the probability corresponding to row i and column j of P, and $\mathbf{s}_{-t} \equiv \mathbf{s}^T \setminus \{s_t\}$. The conditional probability density function, $f(x_t|s_t) = \exp\{-\varepsilon_t^2/(2\sigma^2)\}/\sqrt{2\pi\sigma^2}$, where $\varepsilon_t = x_t - (\bar{\tau}(s_t) - \gamma\overline{by}(s_t)))$ is the discretionary *i.i.d.* tax shock. s_t is drawn from each of these 3-element vectors of conditional probabilities with a uniform random number generator after normalizing by the sum of the elements.

3. Use the importance sampler described in appendix C.3 to draw a new transition matrix, P.
4. Repeat steps 2 and 3 N times.

Household use the average P over the last half of the Gibbs chain as their estimate of the transition matrix, $\hat{P}$, in period T.

C.3 IMPORTANCE SAMPLER This section outlines the importance sampler, which households use to estimate the transition probabilities. The likelihood of observing $\mathbf{s}^T \equiv \{s_t\}_{t=0}^T$ is

$$f(\mathbf{s}^T|\Pi, P) = \left(\prod_{j=1}^{3} \Pi_j(P)^{\mathbf{1}_j}\right)\left(\prod_{i=1}^{3}\prod_{j=1}^{3} p_{ij}^{m_{ij}^o}\right) = \prod_{j=1}^{3} \Pi_j(P)^{\mathbf{1}_j} \prod_{j=1}^{3} p_{1j}^{m_{1j}^o} \prod_{j=1}^{3} p_{2j}^{m_{2j}^o} \prod_{j=1}^{3} p_{3j}^{m_{3j}^o}, \tag{7}$$

where Π_j is the jth element of the stationary distribution, $\mathbf{1}_j \in \{0,1\}$ is an indicator for whether state j is occupied by households at time 0, p_{ij} is the probability corresponding to row i and column j of the transition matrix, and m_{ij}^o is the number of observed transitions from states i to j. Following Geweke (2005), the conjugate prior follows a Dirichlet distribution given by

$$f(P) = \left[\prod_{i=1}^{3} \Gamma\left(\sum_{j=1}^{3} a_{ij}\right) \Big/ \prod_{i=1}^{3}\prod_{j=1}^{3} \Gamma(a_{ij})\right]\left(\prod_{i=1}^{3}\prod_{j=1}^{3} p_{ij}^{a_{ij}-1}\right), \tag{8}$$

where $a_{ij} > 0$ are the shaping parameters of the prior distribution, and Γ is the gamma function. The posterior density is given by the product of the likelihood function, (7), and the prior density, (8). Thus, dropping the constants of proportionality, the posterior density is

$$f(P|\mathbf{s}^T) \propto \left(\prod_{j=1}^{3} \Pi_j(P)^{\mathbf{1}_j}\right)\left(\prod_{i=1}^{3}\prod_{j=1}^{3} p_{ij}^{a_{ij}+m_{ij}^o-1}\right). \tag{9}$$

The number of observed transitions, which are calculated after inferring the current state, s_t, determine the shaping parameters of the posterior distribution. Thus, as time unfolds each household's estimates of the probabilities converge toward the true probabilities.

The posterior distribution, (9), does not correspond to any standard density function we can sample from directly. However, the second component is a product of two independent Dirichlet probability density functions. Consequently, we utilize the following importance sampling algorithm to sample from this distribution. First, sample D draws from a Dirichlet distribution with parameters $\mathbf{a}_i + \mathbf{m}_i$, where $\mathbf{a}_i = (a_{i1}, a_{i2}, a_{i3})$ and $\mathbf{m}_i = (m_{i1}, m_{i2}, m_{i3})$ for $i = \{1,2,3\}$. To accomplish this, we follow the procedure outlined in Gelman et al. (2004, Appendix A). Draws are made from a Gamma distribution with parameters $((\mathbf{a}_i + \mathbf{m}_i)/2, 2)$, and then weighted by the sum of the draws corresponding to each state, i. That is, using the draws, x_{ij}^d, from a Gamma distribution, a draw from a Dirichlet distribution is $\theta_{ij}^d = x_{ij}^d / \sum_{i=1}^{3} x_{ij}^d$. Second, the draws from the Dirichlet distribution are weighted by the coefficient of the posterior distribution, $w_d \equiv \prod_{j=1}^{3} \Pi_j(P_t^d)^{\mathbf{1}_j}$ for all $d \in \{1, \ldots, D\}$, and then divided by the sum of the weights. Formally, the weighting procedure produces an estimate of the transition probabilities (draw from the posterior distribution),

$$\hat{p}_{ij} = \frac{\sum_{d=1}^{D} w_d \theta_{ij}^d}{\sum_{d=1}^{D} w_d}.$$

Alternatively, we could have drawn from the posterior distribution by applying the independence Metropolis-Hastings algorithm or acceptance sampling. Since we use a representative agent model, the efficiency gains from these alternative sampling algorithms is negligible.