Forward Guidance and the State of the Economy*

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ABSTRACT

This paper analyzes forward guidance in a nonlinear model with a zero lower bound (ZLB) on the nominal interest rate. Forward guidance is modeled with news shocks to the monetary policy rule, which capture innovations in expectations from central bank communication about future policy rates. Whereas most studies use quasi-linear models that disregard the expectational effects of hitting the ZLB, we show how the effectiveness of forward guidance nonlinearly depends on the state of the economy, the speed of the recovery, the degree of uncertainty, the policy shock size, and the forward guidance horizon when households account for the ZLB.

Keywords: Forward Guidance; Zero Lower Bound; News Shocks; Nonlinear Solution Method
JEL Classifications: E43; E58; E61

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1 INTRODUCTION

The global economic slowdown in 2008 led many central banks to sharply reduce their policy rates. When rates could not be reduced further, some central banks resorted to unconventional policies, such as forward guidance. Forward guidance refers to central bank communication about future monetary policy, which has many forms including announcements about objectives, contingencies, policy actions, and speeches. Our focus is on communication about the path of future policy rates.[1]

This paper analyzes forward guidance in a nonlinear New Keynesian model with an occasionally binding zero lower bound (ZLB) constraint on the nominal interest rate. Forward guidance is modeled with news shocks to the monetary policy rule similar to Laséen and Svensson (2011).[2] In our model, news shocks are expected future shocks to the monetary policy rule, whereas monetary policy shocks are contemporaneous shocks to the policy rule. The central bank provides forward guidance by communicating the news over a specific horizon, which is represented by a series of expected future monetary policy shocks. The central bank's forward guidance announcement provides news to the market that causes expected future policy rates to fall. Therefore, news shocks are a modeling device for generating innovations in expectations, and they are only effective in periods when the expected policy rate is above zero. News that the central bank intends to lower future policy rates raises inflation, lowers real interest rates, and boosts output over the entire horizon.

We show the effectiveness of forward guidance nonlinearly depends on the state of the economy, the speed of the recovery, the degree of economic uncertainty, the monetary response to inflation, the policy shock size, and the forward guidance horizon. Forward guidance has nonlinear effects because the ZLB constraint restricts the margin for forward guidance to lower expected nominal rates as they approach zero. When nominal rates are at or near the ZLB, we find the stimulus from forward guidance falls as the economy deteriorates, the monetary response to inflation is more aggressive, or households expect a slower recovery because the central bank has less room to lower expected nominal rates. At the ZLB, less uncertainty about future economic conditions increases the probability future nominal rates will stay at zero which limits the ability of policymakers to reduce expected nominal rates as a means of stimulating economic activity. Over longer forward guidance horizons, prospects for an economic recovery are better, which pushes up expected nominal rates and provides the policy with a larger margin to stimulate the economy. The central bank, however, has more difficulty affecting expectations as the horizon is extended. When the total amount of news is fixed and distributed over various horizons, we find the cumulative response of real GDP initially increases but then decreases, which indicates the central bank faces limits on how far forward guidance can extend into the future and continue to add stimulus. While many factors influence the stimulative effect of forward guidance, our results stress that it is crucial to impose the ZLB not just on the current nominal rate but on all expected future nominal rates.

Whereas most studies of forward guidance use quasi-linear models that do not account for the expectational effects of hitting the ZLB, to our knowledge this paper is the first to analyze forward guidance with news shocks using a nonlinear solution.[3] A nonlinear solution enhances our analysis

[1]See the Bank of England (2013) for a discussion of how forward guidance helps the public form more accurate expectations about future central bank policies. See den Haan (2013) for a collection of essays about forward guidance and the International Monetary Fund (2013) for a detailed account of recent unconventional monetary policies.

[2]Gomes et al. (2017) and Milani and Treadwell (2012) estimate unconstrained New Keynesian models that include news shocks in the monetary policy rule. They find news shocks play an important role in matching data. Ben Zeev et al. (2015) and Campbell et al. (2012) develop methods to identify anticipated monetary policy shocks in the data.

[3]We refer to models that impose the ZLB constraint on the nominal interest rate in an otherwise linear framework

of forward guidance in several ways. One, it enables ZLB events to endogenously reoccur, which impacts households' expectations of future policy rates and the central bank's ability to provide economic stimulus. Two, we can assess the impact of forward guidance at the ZLB, near the ZLB, or at any other state of the economy. Three, it allows us to evaluate forward guidance in a setting where changes in economic conditions affect both the probability and expected duration of a ZLB event. For example, a negative demand shock while the ZLB binds reduces a central bank's margin to lower expected policy rates by decreasing the probability of exiting the ZLB. Four, we are able to analyze forward guidance across all possible realizations of shocks, which nonlinearly impact the economy. We also show that failing to incorporate the ZLB constraint into households' expectations causes the model to significantly overstate the stimulative effect of forward guidance.[4]

Campbell et al. (2012) introduce two terms to identify the types of forward guidance: Delphic and Odyssean. Delphic forward guidance is a central bank's forecast of its own policy, which is based on its projections for inflation and real GDP as well as an established policy rule. If the policy rule is known to the public, then Delphic forward guidance on the policy rate path is redundant. One reason to announce forward guidance about the policy rate together with economic forecasts is to clarify the central bank's policy strategy. Odyssean forward guidance is a promise to deviate from the policy rule in the future by setting the policy rate lower than the rule recommends. Recently, central banks have communicated their intention to keep their policy rate at zero longer than their policy rule would suggest. News shocks are a way to model Odyssean forward guidance.

Central banks have recently used both date-based and threshold-based forward guidance. Date-based forward guidance provides information on the intended policy rate path over a fixed period and is often modeled using an interest rate peg. To a modeler, an interest rate peg is a special case of our news shock approach, where the central bank provides news that it intends to fix the policy rate for a set number of periods. We compare our approach to modeling forward guidance to an interest rate peg. The peg generates increasingly larger impact effects on output as the horizon is extended because it gives the central bank a growing ability to affect expected future interest rates.

With threshold-based forward guidance, the central bank agrees to maintain a policy rate until a specific event occurs. For example, the central bank might announce it intends to keep its policy rate at zero until the unemployment rate falls below a certain value. Our news shock approach is similar to threshold-based forward guidance because it allows the policy rate to endogenously respond to economic conditions until the objectives for output and inflation have been met. While the news shocks are Odyssean, the endogenous response of monetary policy to economic conditions is Delphic because households know the central bank's rule and use it to forecast future policy rates.

There are four reasons why we advocate using news shocks instead of an interest rate peg to model forward guidance. One, news shocks are more flexible since an interest rate peg corresponds to a specific sequence of anticipated shocks. Two, an interest rate peg produces a degenerate distribution for the policy rate that contradicts recent survey and options data. In our model, the distribution for every future nominal interest rate depends on the distribution of future economic outcomes. Three, households never expect the central bank to adjust its forward guidance policies to economic conditions under an interest rate peg, which is inconsistent with the threshold-based nature of recent forward guidance. With news shocks, households' expectations incorporate the

as quasi-linear models. A few papers that use this modeling approach include Carlstrom et al. (2015), Christiano et al. (2011), Eggertsson and Woodford (2003), Erceg and Lindé (2014), Guerrieri and Iacoviello (2015), Jung et al. (2005).

[4]Other papers also show there are important drawbacks to using a model that does not incorporate the ZLB constraint in households' expectations (see Gavin et al. (2015), Gust et al. (2013), and Richter and Throckmorton (2016)).

possibility the policy rate could rise due to improving economic conditions. Four, an interest rate peg does not separate the effects of additional news from a longer horizon because extending a peg is analogous to providing increasingly large news shocks. For those reasons, we believe news shocks provide the sophistication necessary to accurately assess the effects of forward guidance.

Other papers examine the effectiveness of forward guidance in an economy with a binding ZLB constraint through the perspective of optimal monetary policy under commitment (i.e., a promise to implement a specific policy regardless of changes in future economic conditions).[5] Eggertsson and Woodford (2003) and Jung et al. (2005) solve for the optimal commitment policy assuming the policy rate initially equals zero, but once it starts to rise, it cannot return to the ZLB. They find the optimal policy is to maintain a policy rate equal to zero even after the natural real interest rate rises. Such a policy generates higher future inflation and lowers the real interest rate, which moderates the declines in output and inflation that occur at the ZLB. Levin et al. (2010) show the optimal policy stabilizes the economy after small shocks but not after large and persistent shocks. In that situation, they argue central banks should use other unconventional policies, such as quantitative easing, to stabilize the economy. Adam and Billi (2006) relax the assumption that the policy rate initially equals zero by allowing the ZLB constraint to occasionally bind. They find the optimal commitment policy is to respond more aggressively to shocks that decrease output and inflation.[6]

There is also work on forward guidance outside the optimal policy literature. Del Negro et al. (2015) use a log-linear New Keynesian model to show that extending the forward guidance horizon causes the model to overpredict the actual increases in output and inflation. They call that result the "forward guidance puzzle" and show that introducing finitely lived agents provides a potential resolution to the puzzle. Several other papers offer alternative explanations. For example, McKay et al. (2016) introduce uninsurable income risk and borrowing constraints, Kiley (2016) considers a model with sticky information rather than sticky prices, De Graeve et al. (2014) and Haberis et al. (2014) account for imperfect credibility, and Caballero and Farhi (2014) develop a model where the ZLB binds due to a safety trap—a shortage of safe assets—instead of a demand-side shock.

We emphasize the ZLB constraint on current and future policy rates and the state of the economy as a way of explaining the forward guidance puzzle. In our model, demand shocks push the policy rate to its ZLB. The size of those shocks and whether news shocks occur determine how long the policy rate remains at zero. As demand falls, the ZLB constraint further limits the stimulative effect of forward guidance by preventing future policy rates from declining. Although most New Keynesian models overpredict the stimulative effect of forward guidance, our results are consistent with the estimates in D'Amico and King (2015). They find anticipated reductions in the policy rate boost output over horizons up to four quarters but have much weaker effects over longer horizons.

The rest of the paper is organized as follows. Section 2 provides a post-financial crisis account of Federal Open Market Committee (FOMC) forward guidance in its policy statements. Section 3 describes our theoretical model. Sections 4 and 5 show the stimulative effects of forward guidance across horizons up to 10 quarters. Section 6 conducts case studies of recent FOMC forward guidance and uses our key findings to explain the effects of that communication. Section 7 concludes.

[5] Krugman (1998) was the first to argue that the central bank can mitigate the effects of the ZLB by promising to allow prices to rise. Reifschneider and Williams (2000) develop the merits of that argument in a dynamic model.

[6] Werning (2011) shows it is optimal to commit to higher future inflation when the ZLB binds in a continuous-time model. Adam and Billi (2007) find discretionary policy is unable to generate the higher inflation necessary to offset the adverse effects of the ZLB. English et al. (2015) show that introducing threshold-based forward guidance into the monetary policy rule generates outcomes closer to the optimal commitment policy. Coenen and Warne (2014) find date-based forward guidance increases the risk of price instability, but a threshold on inflation can reduce that risk.

2 RECENT FEDERAL RESERVE FORWARD GUIDANCE

There are two ways the Fed communicates information about future policy rates. One, it releases the individual forecasts of the FOMC members every quarter. Two, it provides forward guidance about the future federal funds rate in its policy statements and has consistently done so since 2008.

At the December 16, 2008 meeting, the FOMC decided to target a range for the federal funds rate of 0% to 0.25% and announced it would likely remain at that low level for "some time." The FOMC continued to use similar language until its August 9, 2011 statement, which said that low range was likely warranted "at least through mid-2013." That announcement was the FOMC's first use of date-based forward guidance, and it had a sizable effect on expected future interest rates.

The FOMC's forward guidance was modified in two ways after the January 25, 2012 meeting. One, the FOMC said the federal funds rate was expected to remain at zero "at least through late 2014," which was a six quarter increase. Two, the FOMC expressed a more pessimistic economic outlook and indicated the projected path for the federal funds rate was conditional on that outlook, which suggests the FOMC was already projecting a much later date for raising its policy rate. Therefore, the forward guidance provided in the January statement was likely viewed as Delphic.

Despite the forward guidance extension, the economy continued to disappoint policymakers, which motivated the FOMC to amend its statement in a couple ways after the September 13, 2012 meeting. One, the FOMC stated that "exceptionally low levels of the federal funds rate are likely to be warranted at least through mid-2015," which was a 2-quarter extension to the time the policy rate was expected to remain at zero. Two, the FOMC noted that "a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens." That precise language conveys Odyssean forward guidance because without it, the expectation is that the FOMC would raise its policy rate as the economy improves. The FOMC statement also included information about business spending that likely lowered real GDP growth forecasts, which suggests the change in forward guidance may have also been viewed as Delphic.

On December 12, 2012, the FOMC changed its forward guidance from date-based to threshold-based for the first time. The statement said "this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above $6\text{-}1/2$ percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored." FOMC participants' forecasts indicated the unemployment rate would likely hit 6.5% in mid-2015. Therefore, the statement was not intended to change expectations about when the policy rate would rise, but rather to emphasize that any policy rate changes are conditional on inflation expectations and labor market conditions. The phrase "at least as long as" suggests the unemployment rate threshold would not automatically trigger the FOMC to raise its policy rate.

Over the next year, the labor market continued to improve, and it became apparent the unemployment rate might fall below the 6.5% threshold sooner than expected. On December 18, 2013, the FOMC redrafted its forward guidance by stating "…it likely will be appropriate to maintain the current target range for the federal funds rate well past the time that the unemployment rate declines below $6\text{-}1/2$ percent." The change in language from "at least as long as" to "well past" may have been viewed as Odyssean because it implied the policy rate would remain near zero even though stronger economic conditions would normally trigger the FOMC to raise its policy rate.

The FOMC's state-contingent forward guidance continued to evolve in 2014 and 2015. The March 19, 2014 statement said the FOMC would likely target a low range for the federal funds rate for a "considerable time after the asset purchase program ends." The FOMC's statement was revised on January 28, 2015 to say "it can be patient in beginning to normalize" rates. By June 17, 2015, future rate increases appeared imminent as 15 of the 17 committee members were forecasting a rate increase in 2015. The FOMC finally raised its policy rate by 25 basis points on December 16, 2015, which was the first increase since June 2006. The high likelihood of remaining in a low interest rate environment emphasizes the importance of analyzing forward guidance not only at the ZLB but near the ZLB, especially since the FOMC has said "economic conditions may, for some time, warrant keeping the target federal funds rate below levels the Committee views as normal."[7]

3 Economic Model

This section describes our model. Forward guidance enters through news shocks to the monetary policy rule. Our presentation is brief because, other than the news shocks, we use a standard New Keynesian model that has monopolistically competitive firms and quadratic price adjustment costs.

3.1 Households A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t [\log c_t - \chi n_t^{1+\eta}/(1+\eta)]$, where c is consumption, n is labor hours, b is the real value of a 1-period nominal bond, $1/\eta$ is the Frisch elasticity of labor supply, E_0 is the mathematical expectations operator conditional on information available in period 0, $\tilde{\beta}_0 \equiv 1$, and $\tilde{\beta}_t = \prod_{i=1}^{t} \beta_i$ for $t \geq 1$. Following Eggertsson and Woodford (2003), the discount factor follows $\beta_t = (1-\rho_\beta)\bar{\beta} + \rho_\beta \beta_{t-1} + \upsilon_t$, where $\bar{\beta}$ is the steady-state value, $0 \leq \rho_\beta < 1$, and $\upsilon_t \sim \mathbb{N}(0, \sigma_\upsilon^2)$.

The household's choices are constrained by $c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t$, where π_t is the gross inflation rate, w_t is the real wage rate, i_t is the gross nominal interest rate, and d_t are the dividends from intermediate firms. The optimality conditions to the household's problem imply

$$w_t = \chi n_t^{\eta} c_t, \tag{1}$$
$$1 = i_t E_t[\beta_{t+1}(c_t/c_{t+1})/\pi_{t+1}]. \tag{2}$$

3.2 Firms The production sector consists of monopolistically competitive intermediate goods firms and a final goods firm. Intermediate firm $f \in [0,1]$ produces a differentiated good, $y_t(f)$, according to $y_t(f) = n_t(f)$, where $n_t(f)$ is the labor used by firm f. Each intermediate firm chooses its labor input to minimize operating costs, $w_t n_t(f)$, subject to its production function. The final goods firm purchases $y_t(f)$ from each intermediate firm to produce the final good, $y_t \equiv [\int_0^1 y_t(f)^{(\theta-1)/\theta} df]^{\theta/(\theta-1)}$, according to a Dixit and Stiglitz (1977) aggregator, where $\theta > 1$ is the elasticity of substitution between the intermediate goods. The demand function for intermediate inputs is $y_t(f) = (p_t(f)/p_t)^{-\theta} y_t$, where $p_t = [\int_0^1 p_t(f)^{1-\theta} df]^{1/(1-\theta)}$ is the price of the final good.

Following Rotemberg (1982), each firm faces a price adjustment cost, $adj_t(f)$. Using the functional form in Ireland (1997), $adj_t(f) = \varphi[p_t(f)/(\bar{\pi} p_{t-1}(f)) - 1]^2 y_t/2$, where $\varphi \geq 0$ scales the size of the adjustment costs and $\bar{\pi}$ is the steady-state gross inflation rate. Real dividends are then given by $d_t(f) = (p_t(f)/p_t) y_t(f) - w_t n_t(f) - adj_t(f)$. Firm f chooses its price, $p_t(f)$, to maximize the expected discounted present value of real dividends, $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t (c_0/c_t) d_t(f)$. In a

[7]Forward guidance has also been used by the Bank of Canada, Bank of England, European Central Bank, Bank of Japan, Reserve Bank of New Zealand, Norges Bank, and the Riksbank. See Andersson and Hofmann (2010), Filardo and Hofmann (2014), Kool and Thornton (2015), Moessner and Nelson (2008), Moessner et al. (2016), Svensson (2011, 2015), and Swanson and Williams (2014) for an overview of the various policies and analysis of their effects.

symmetric equilibrium, all firms make identical decisions and the optimality condition implies

$$\varphi\left(\frac{\pi_t}{\bar{\pi}}-1\right)\frac{\pi_t}{\bar{\pi}} = (1-\theta)+\theta w_t+\varphi E_t\left[\beta_{t+1}\frac{c_t}{c_{t+1}}\left(\frac{\pi_{t+1}}{\bar{\pi}}-1\right)\frac{\pi_{t+1}}{\bar{\pi}}\frac{y_{t+1}}{y_t}\right]. \tag{3}$$

Without price adjustment costs, the gross markup of price over marginal cost equals $\theta/(\theta-1)$.

3.3 Central Bank and Forward Guidance

The policy rate is set according to

$$\begin{gathered} i_t = \max\{\underline{\imath}, i_t^*\}, \qquad i_t^* = \bar{\imath}(\pi_t/\bar{\pi})^{\phi_\pi}(y_t/\bar{y})^{\phi_y}\exp(x_t), \\ x_t \equiv \textstyle\sum_{j=0}^{q}\alpha_j\varepsilon_{t-j}, \quad \sum_{j=0}^{q}\alpha_j = 1, \end{gathered} \tag{4}$$

where $\underline{\imath}$ is the lower bound on the nominal interest rate, i_t^* is the notional interest rate (i.e., the rate the central bank would set if it was unconstrained), $\bar{\pi}$ and $\bar{\imath}$ are the steady-state inflation and nominal interest rates, ϕ_π and ϕ_y are the policy responses to the inflation and output gaps, $\varepsilon_t \sim \mathbb{N}(0,\sigma_\varepsilon^2)$ is a monetary policy shock, α_j is the weight on the shock to the nominal interest rate j periods ahead, and $q \geq 0$ is the forward guidance horizon. For example, when $(\alpha_0,\alpha_1,\ldots,\alpha_q) = (1,0,\ldots,0)$, the shock is unanticipated (no forward guidance) and when $(\alpha_0,\alpha_1,\ldots,\alpha_q) = (0,0,\ldots,1)$, the shock is anticipated in q periods (q-period forward guidance).

The constraint on the α's holds the total weight on the news shocks constant across various forward guidance horizons, which is crucial for two reasons. One, it allows us to isolate the effect of a longer horizon from additional news. Without the restriction, it would be impossible to identify the effect of an increase in q, because the forward guidance extension would also increase the total amount of news and stimulate the economy. Two, it places a restriction on the total amount the central bank can affect expected future interest rates; otherwise, the central bank would have a growing ability to create innovations in expectations by lengthening the forward guidance horizon.

3.4 Equilibrium

The resource constraint is $c_t = y_t - adj_t \equiv y_t^{gdp}$, where y_t^{gdp} includes the value added by intermediate firms, which is their output minus price adjustment costs. Thus, y_t^{gdp} represents real GDP in the model. A competitive equilibrium consists of sequences of quantities, $\{c_t, n_t, y_t, b_t\}_{t=0}^{\infty}$, prices, $\{w_t, i_t, \pi_t\}_{t=0}^{\infty}$, and discount factors, $\{\beta_t\}_{t=0}^{\infty}$, that satisfy the household's and firms' optimality conditions, (1)-(3), the monetary policy rule, (4), the production function, $y_t = n_t$, the bond market clearing condition, $b_t = 0$, the discount factor process, and the resource constraint, given the initial conditions, β_{-1} and $\{\varepsilon_{-j}\}_{j=0}^{q}$, and sequences of shocks, $\{\varepsilon_t, \upsilon_t\}_{t=1}^{\infty}$.

3.5 Calibration

We calibrate our model at a quarterly frequency to match moments in U.S. data from 1983Q1 to 2014Q4 (i.e., the post-Volcker disinflation period). The parameters are summarized in table 1. The steady-state discount factor, $\bar{\beta}$, is set to 0.9957, which equals the average ratio of the GDP implicit price deflator inflation rate to the 3-month T-bill rate. The Frisch elasticity of labor supply, $1/\eta$, is set to 3, which matches the macro estimate in Peterman (2016). The leisure preference parameter, χ, is calibrated so steady-state labor equals $1/3$ of the available time. The elasticity of substitution between intermediate goods, θ, is calibrated to 6, which corresponds to a 20% average markup of price over marginal cost and matches the commonly-used estimate in Christiano et al. (2005). The price adjustment cost parameter, φ, is set to 160, which is equivalent to a Calvo price duration of about 6 quarters in a linear model. The lower bound on the nominal interest rate, $\underline{\imath}$, is set to 1.00022, which is the average 3-month T-bill rate from 2009Q1 to 2014Q4.

Steady-State Discount Factor	$\bar{\beta}$	0.9957	Nominal Interest Rate Lower Bound	$\underline{\imath}$	1.00022
Frisch Elasticity of Labor Supply	$1/\eta$	3	Monetary Policy Response to Inflation	ϕ_π	2
Elasticity of Substitution between Goods	θ	6	Monetary Policy Response to Output	ϕ_y	0.08
Rotemberg Adjustment Cost Coefficient	φ	160	Discount Factor Persistence	ρ_β	0.87
Steady-State Labor	$\bar{n}$	0.33	Discount Factor Standard Deviation	σ_υ	0.00225
Steady-State Inflation Rate	$\bar{\pi}$	1.0057	Monetary Policy Shock Standard Deviation	σ_ε	0.003

Table 1: Calibrated parameters.

The steady-state inflation rate, $\bar{\pi}$, is calibrated to 1.0057 to match the average GDP deflator inflation rate. Using the estimates from Smets and Wouters (2007), we set the monetary response to inflation, ϕ_π, equal to 2 and the response to output, ϕ_y, equal to 0.08. An iterative process is used to pin down the weights in the news process (i.e., the α's). Given a specific forward guidance horizon and a guess for the α's, we first solve the nonlinear model. Next, a nonlinear solver is used to determine an updated set of α's that minimize the effect of a policy shock on the nominal interest rate in periods t to $t+q-1$. The iterative process continues until the maximum change is below 10^{-4}. We also solve the model with no forward guidance and q-quarter forward guidance.

The persistence of the discount factor, ρ_β, equals 0.87 and the standard deviation of the shock, σ_υ, equals 0.00225, which are close to the estimates in Gust et al. (2013). The standard deviation of the monetary policy shock, σ_ε, is set to 0.003. We chose these parameters to match volatilities in the data and the length of time people *expected* the ZLB to bind, rather than the duration of the current ZLB episode. In the data, the annualized standard deviations of quarter-over-quarter percent changes in real GDP, the GDP deflator inflation rate, and the 3-month T-bill rate are 2.58%, 0.99%, and 2.79%, respectively, per year. To compare our model to those values, we run $10{,}000$ simulations that are each 128 quarters long (i.e., the same length as our data). We then compute the median standard deviations of real GDP growth, the inflation rate, and the nominal interest rate. Those values and their 95% credible intervals are 2.45% $(1.92\%, 3.67\%)$, 1.07% $(0.74\%, 1.63\%)$, and 2.29% $(1.83\%, 2.90\%)$, respectively, per year. The median standard deviations in the model are near their historical averages, and all three credible intervals contain the values in the data.

Prior to the FOMC's August 2011 date-based forward guidance, survey data indicated the 3-month T-bill rate was not expected to remain near zero for very long. Blue Chip consensus forecasts from 2009 and 2010 reveal that the 3-month T-bill rate was expected to exceed 0.5% within three quarters. In our model, a ZLB event lasts an average of 2.12 quarters when the economy is initialized at its steady state but rises to 3.10 quarters when it is initialized at a notional interest rate that is consistent with estimates during and immediately after the Great Recession. Therefore, our calibration produces ZLB events with a similar average duration to what was expected prior to the FOMC's forward guidance. It is also possible for the model to generate much longer ZLB events.

3.6 SOLUTION METHOD The model is solved using the policy function iteration algorithm described in Richter et al. (2014), which is based on the theoretical work on monotone operators in Coleman (1991). This method discretizes the state space and iteratively solves for updated policy functions until the tolerance criterion is met. We use linear interpolation to approximate future variables, since it accurately captures the kink in the policy functions, and Gauss-Hermite quadrature to numerically integrate. See appendix C for a formal description of the algorithm.[8]

[8] Benhabib et al. (2001) show that models with a ZLB constraint have two steady-state equilibria. See Gavin et al. (2015) for a discussion of the equilibrium that our algorithm converges to in both a deterministic and stochastic model.

4 ONE-QUARTER HORIZON RESULTS

This section first quantifies the stimulative effect of forward guidance over a 1-quarter horizon. We then show the importance of the ZLB constraint, the degree of uncertainty, the monetary response to inflation, the state of the economy, the size of the news shock, and the speed of the recovery.

4.1 EFFECTS OF FORWARD GUIDANCE Figure 1 plots the decision rules for real GDP, the inflation rate, and the current and expected future nominal interest rates as a function of the monetary policy shock, $\hat{\varepsilon}_t$.[9] The time subscript is the period households learn about the shock and not necessarily the period the shock impacts the economy. If the central bank provides no forward guidance, then $\hat{\varepsilon}_t$ is an unanticipated monetary policy shock that impacts the economy in period t. When the central bank provides 1-quarter forward guidance, $\hat{\varepsilon}_t$ is a news shock that households learn about in period t but does not impact the economy until period $t+1$. Thus, a news shock creates an innovation in the expected nominal interest rate, which can be directly mapped to changes in forecasts that occur after an FOMC statement is released. We quantify the effects of 1-quarter forward guidance by comparing the differences in forecasts before and after the policy announcement. The vertical axis displays the marginal effect of a monetary policy shock relative to when there is no shock. For example, a 1-quarter news shock of $\hat{\varepsilon}_t = -0.25$ lowers the expected nominal interest rate by roughly 0.1 percentage points and raises real GDP by about 0.1% relative to when $\hat{\varepsilon}_t = 0$.

We focus on a cross section of the decision rules where the initial notional interest rate equals zero because it produces the largest stimulative effect of forward guidance when the central bank is constrained by the ZLB. The notional rate equals zero when the discount factor is 0.61% above its steady state. The elevated discount factor signifies an increased desire by households to save, which lowers inflation and real GDP. Households, however, expect the discount factor to decline over time. If no forward guidance is provided, that belief raises the expected nominal interest rate.

When $(\alpha_0, \alpha_1) = (1, 0)$ (solid line), the central bank provides no forward guidance, so $\hat{\varepsilon}_t$ represents an unanticipated policy shock. If $\hat{\varepsilon}_t > 0$, then the shock contracts economic activity by raising the current nominal interest rate and lowering inflation and real GDP. The expected nominal interest rate is unaffected since the shock is serially uncorrelated. If, however, $\hat{\varepsilon}_t < 0$, then monetary policy has no impact on the nominal interest rate since it is already at its ZLB. Thus, the decision rules remain at zero when $\hat{\varepsilon}_t < 0$ since conventional monetary policy is ineffective.

When $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line), the central bank provides households with 1-quarter forward guidance. The light-shaded regions represent the marginal effects of that policy. The news in period t that an expansionary shock will occur in period $t+1$ leads to a downward revision in the expected nominal interest rate. That expectational effect stimulates real GDP, which raises both the inflation and nominal interest rates—what we refer to as *feedback effects*—even though the discount factor remains at the minimum value necessary for the ZLB to bind. The maximum amount the expected nominal interest rate can decline is the difference between the expected rate in the absence of forward guidance and the ZLB, which is represented by a horizontal dashed line.

The feedback effect on the nominal interest rate from 1-quarter forward guidance is counterfactual to recent FOMC forward guidance, and it would show up in expected nominal rates over longer horizons. In reality, the FOMC did not communicate an increase in current or future nominal interest rates. In our model, the central bank can eliminate the feedback effect by redistributing

[9] In our results, a hat denotes a percent change and a tilde denotes a percentage point difference between net rates.

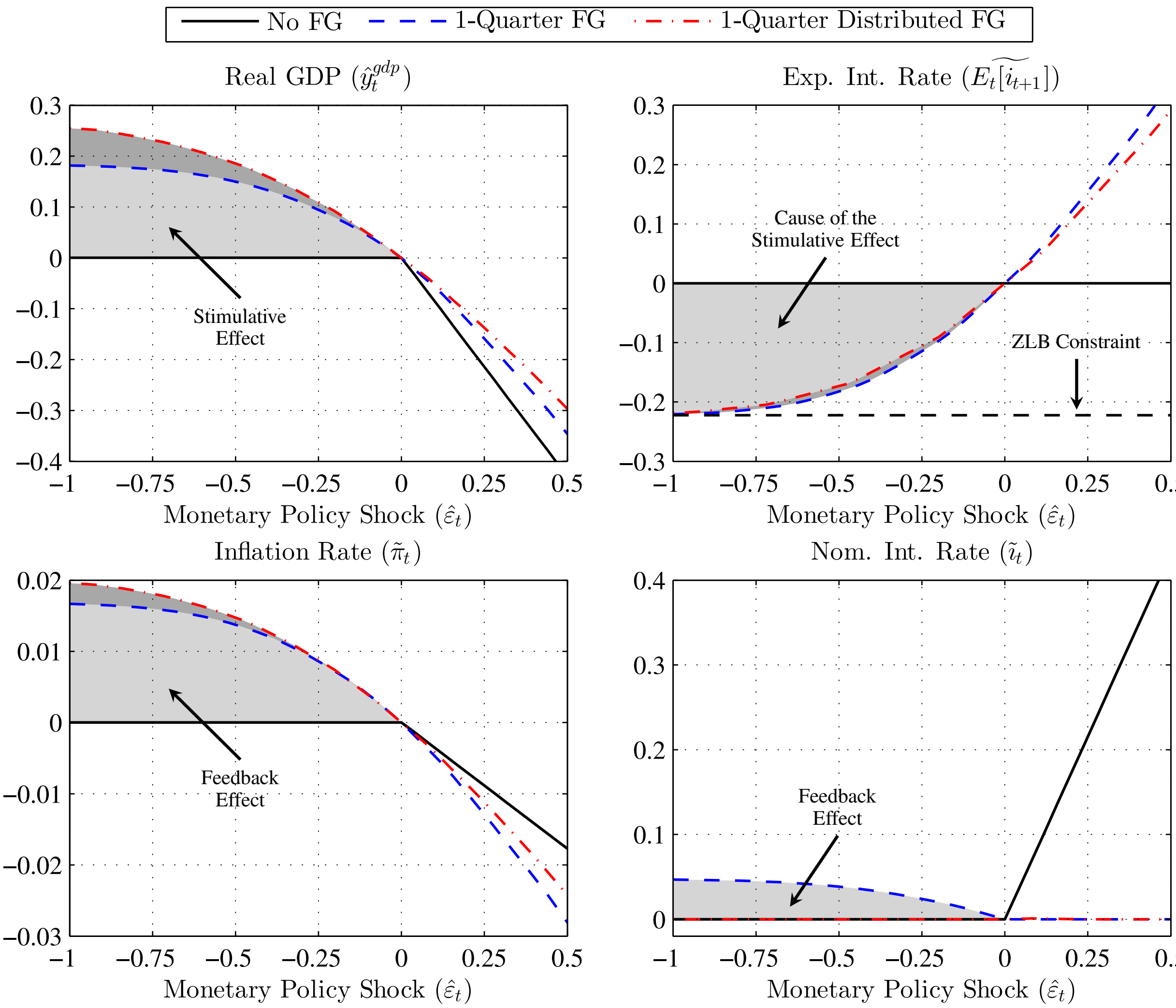


Figure 1: Decision rules as a function of the monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line); 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line); and 1-quarter distributed forward guidance, $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line). In this cross section, the initial notional interest rate equals zero.

the weights on the policy shock, while holding the total weight fixed. The unique policy that precisely eliminates the feedback effect is $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line), which we refer to as 1-quarter distributed forward guidance. In that case, just enough of the weight is taken from the 1-quarter ahead news shock, α_1, and placed on the unanticipated shock, α_0, so the current nominal rate remains at zero. Note, however, that the feedback effects would be much smaller if we initialized the economy at a negative notional rate and nonexistent given a deep enough recession.

Expansionary news shocks under both types of 1-quarter forward guidance have diminishing positive impacts on real GDP as the size of the shock increases. For example, a -0.5% news shock under 1-quarter forward guidance increases real GDP by 0.15 percentage points, whereas a -1% news shock raises real GDP by 0.18 percentage points. Thus, doubling the size of the news shock only leads to a small additional increase in real GDP. The small marginal effect occurs because a larger expansionary policy shock increases the likelihood that next period's nominal interest rate will fall to its ZLB, which is evident from the decision rule for the expected nominal interest rate.

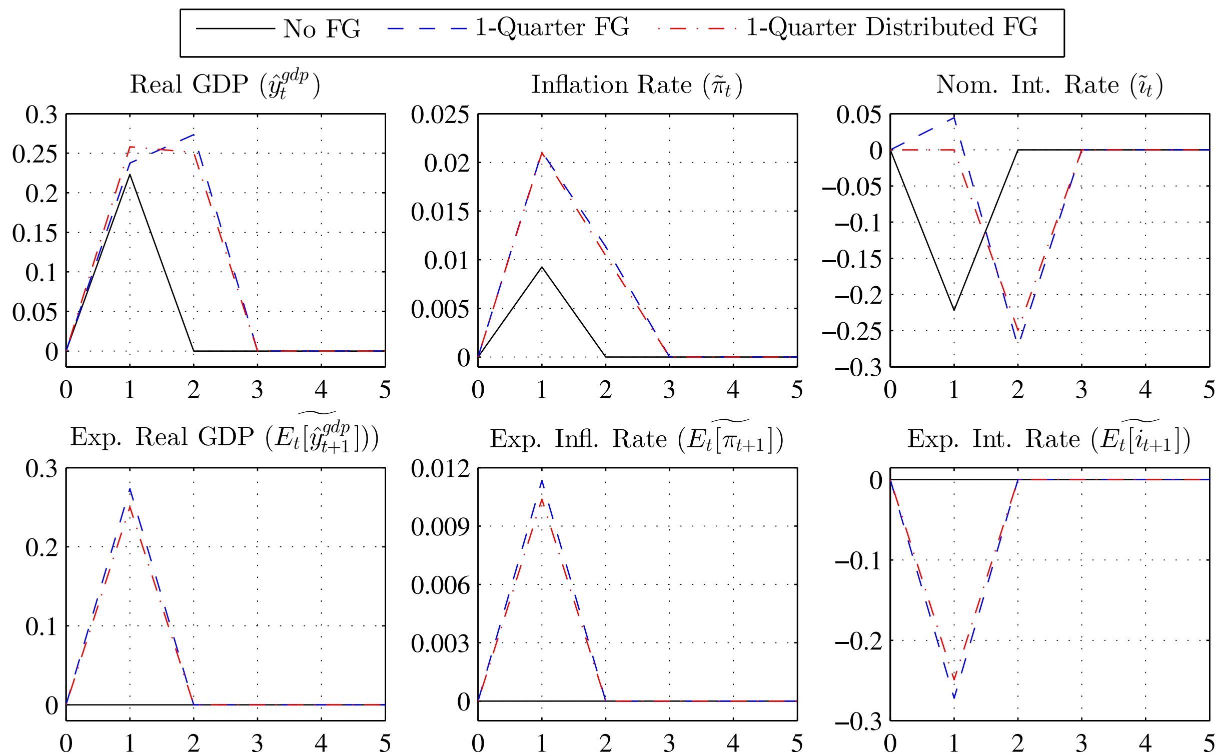


Figure 2: Generalized impulse responses to a -0.5% monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$ (solid line); 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$ (dashed line); and 1-quarter distributed forward guidance, $(\alpha_0, \alpha_1) = (0.13, 0.87)$ (dash-dotted line). Each simulation is initialized at a notional rate equal to zero.

Another way to examine forward guidance is with generalized impulse response functions (GIRFs) following Koop et al. (1996). GIRFs are based on simulations that are consistent with households' expectations. The benefit of GIRFs is they show the dynamic effects of a shock, whereas decision rules show the impact effects for a range of shocks. Figure 2 plots the responses to a -0.5% monetary policy shock at the ZLB with no forward guidance (solid line), 1-quarter forward guidance (dashed line), and 1-quarter distributed forward guidance (dash-dotted line). To compute the GIRFs, we calculate the mean of $100{,}000$ simulations conditional on random shocks. We then calculate a second mean from a new set of $100{,}000$ simulations, but this time the random policy shock in the first quarter of each simulation is replaced with a -0.5% shock. The GIRFs are the percentage change (or difference in rates) between the two means. Each simulation is initialized at a zero notional rate. See appendix D for more details on how we calculate the GIRFs.

In each simulation, households learn about the monetary policy shock in period 1. With no forward guidance, the shock is unanticipated and occurs in period 1. With 1-quarter forward guidance, households receive news in period 1 about a policy shock that will hit in period 2. The combination of a zero notional rate in period 0 and a mean reverting discount factor causes the period 1 nominal interest rate to rise above its ZLB in 59% of the simulations without a monetary policy shock. Therefore, an unanticipated expansionary policy shock [$(\alpha_0, \alpha_1) = (1, 0)$, solid line] in period 1 reduces the nominal rate in most simulations, so the shock on average is stimulative.

A -0.5% 1-quarter forward guidance shock [$(\alpha_0, \alpha_1) = (0, 1)$, dashed line] lowers the expected nominal interest rate and raises expected real GDP and expected inflation in period 2.

Those changes boost real GDP in period 1. Therefore, 1-quarter forward guidance stimulates the economy over the entire forward guidance horizon. The feedback effect increases the nominal interest rate by 0.04% in period 1. Our specification of 1-quarter distributed forward guidance $[(\alpha_0, \alpha_1) = (0.13, 0.87)$, dash-dotted line] shifts just enough weight to the unanticipated shock to completely offset the feedback effect from the period 1 news shock, so the shock has no effect on the nominal interest rate in period 1. As a result, real GDP rises 0.02 percentage points more on impact with distributed forward guidance, while the response in period 2 is only slightly smaller.

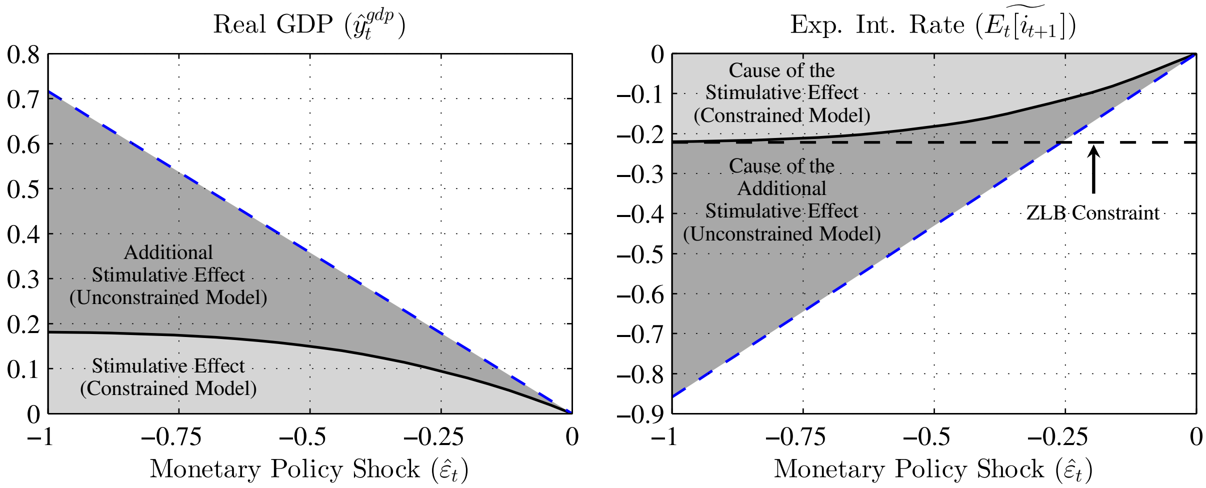


Figure 3: Comparison of decision rules with (solid line) and without (dashed line) a ZLB constraint given 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$. In this cross section of the decision rules, the initial notional rate equals zero.

4.2 IMPORTANCE OF THE ZLB CONSTRAINT The previous section shows forward guidance becomes progressively less stimulative as the expected nominal interest rate approaches zero. Essentially, the ZLB constraint truncates the distribution for the future nominal interest rate at zero, which limits the central bank's ability to lower its expected value. Figure 3 compares the effects of 1-quarter forward guidance with (light-shaded area) and without (dark-shaded area) a ZLB constraint under the assumption that the initial notional interest rate equals zero. That assumption enables us to analyze the effects of the ZLB constraint when the expected nominal rate is near zero. We show the effects of 1-quarter forward guidance rather than distributed forward guidance, so the stimulative effect is only due to changes in the expected nominal interest rate. As in figure 1, the vertical axis measures the marginal effect of the news shock relative to when there is no shock.

Figure 3 reveals the stimulative effect of forward guidance is overstated when the model does not contain a ZLB constraint and the expected nominal interest rate is near or below zero. For example, a -0.5% (-1%) news shock in the constrained model reduces the expected nominal interest rate by 18 (22) basis points and increases real GDP by 0.15 (0.18) percentage points. The same shock in the unconstrained model pushes down the expected nominal rate by 43 (86) basis points and raises real GDP by 0.36 (0.72) percentage points. In that example, the expected nominal rate is below its ZLB, but an overstatement of real GDP also transpires when the expected rate is positive but near zero because part of the distribution for the future nominal rate is negative. The same overstatement would occur if the ZLB constraint is imposed when simulating the model but not when solving it. Since the constraint only affects the current nominal rate when simulating the model and the stimulative effect is entirely driven by the change in the expected nominal rate, it is essential to include the constraint when solving the model to constrain all expected future rates.

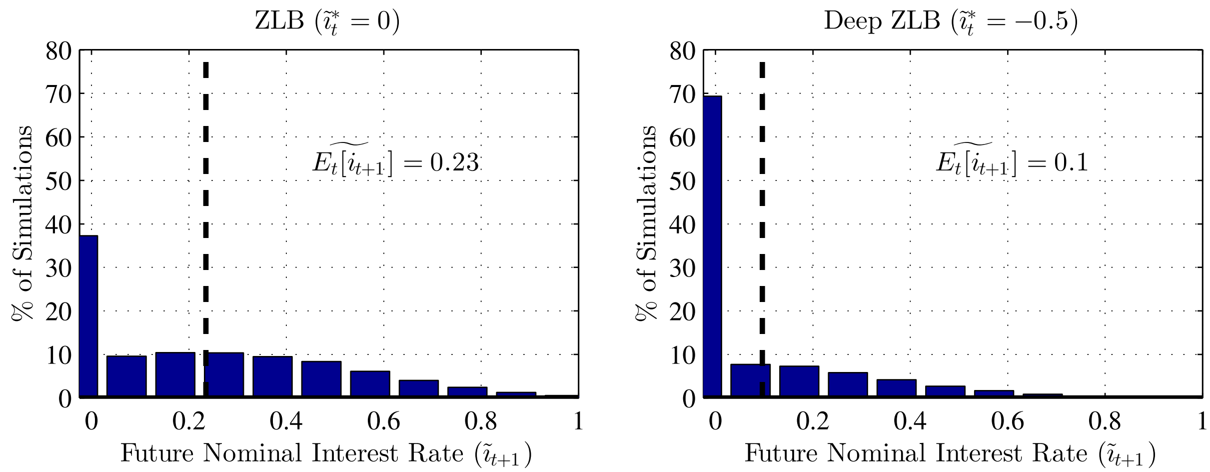


Figure 4: Histograms of the simulated values of next quarter's nominal interest rate without forward guidance. The simulations are initialized at two alternative notional interest rates: $\tilde{i}_t^* = 0$ (left panel) and $\tilde{i}_t^* = -0.5$ (right panel).

4.3 STATE OF THE ECONOMY This section shows how a weak economy can render forward guidance less effective by examining different initial states of the economy. Figure 4 plots histograms of the simulated values of next quarter's nominal interest rate without forward guidance. The dashed lines represent the expected nominal interest rates. The simulations are initialized at two alternative notional interest rates: $\tilde{i}_t^* = 0$ (left panel) and $\tilde{i}_t^* = -0.5$ (right panel). These histograms reveal the distribution for the future nominal interest rate becomes more skewed toward zero as the initial notional rate becomes more negative. For example, 37% (69%) of simulations for $\tilde{i}_{t+1}$ are constrained by the ZLB when $\tilde{i}_t^* = 0$ ($\tilde{i}_t^* = -0.5$), which causes the expected nominal rate to equal 0.23% (0.10%). That is, a weaker economy skews a larger fraction of the future nominal interest rate distribution towards the ZLB, which lowers the expected nominal rate. The lower expected value means forward guidance has a smaller margin to stimulate demand. Since estimates of the notional rate were well below zero during and immediately after the Great Recession, those results provide one key reason why recent forward guidance likely had a limited economic effect.[10]

GIRFs are a practical tool to show how the stimulative effect of forward guidance is influenced by the state of the economy. Figure 5 displays generalized impulse responses to two different types of -0.5% monetary policy shocks: an unanticipated shock (left panels) and a 1-quarter distributed forward guidance shock (right panels). The effect of each shock is examined given four alternative initial notional interest rates: (1) $\tilde{i}_0^* = 1$ (solid line) represents an economy at its steady state; (2) $\tilde{i}_0^* = 0.25$ (dashed line) is a low policy rate that is consistent with the FOMC's June 2015 forecast for 2016; (3) $\tilde{i}_0^* = 0$ (circle markers) denotes an economy that is just weak enough so the ZLB binds (i.e., the same value used in earlier figures); and (4) $\tilde{i}_0^* = -0.5$ (triangle markers) represents an economy in a severe recession where the policy rate is constrained by the ZLB, which is based on its estimated value during the Great Recession. In each case, the weights on the 1-quarter distributed forward guidance shock (i.e., α_0 and α_1) are set so that monetary policy does not affect the nominal interest rate in period 1 (i.e., the feedback effect is eliminated). A policy that does not generate feedback effects on the nominal rate is consistent with recent FOMC forward guidance.

There are two important takeaways from our simulations. One, monetary policy shocks become

[10]See Bauer and Rudebusch (2016), Krippner (2013), and Wu and Xia (2016) for estimates of the notional rate.

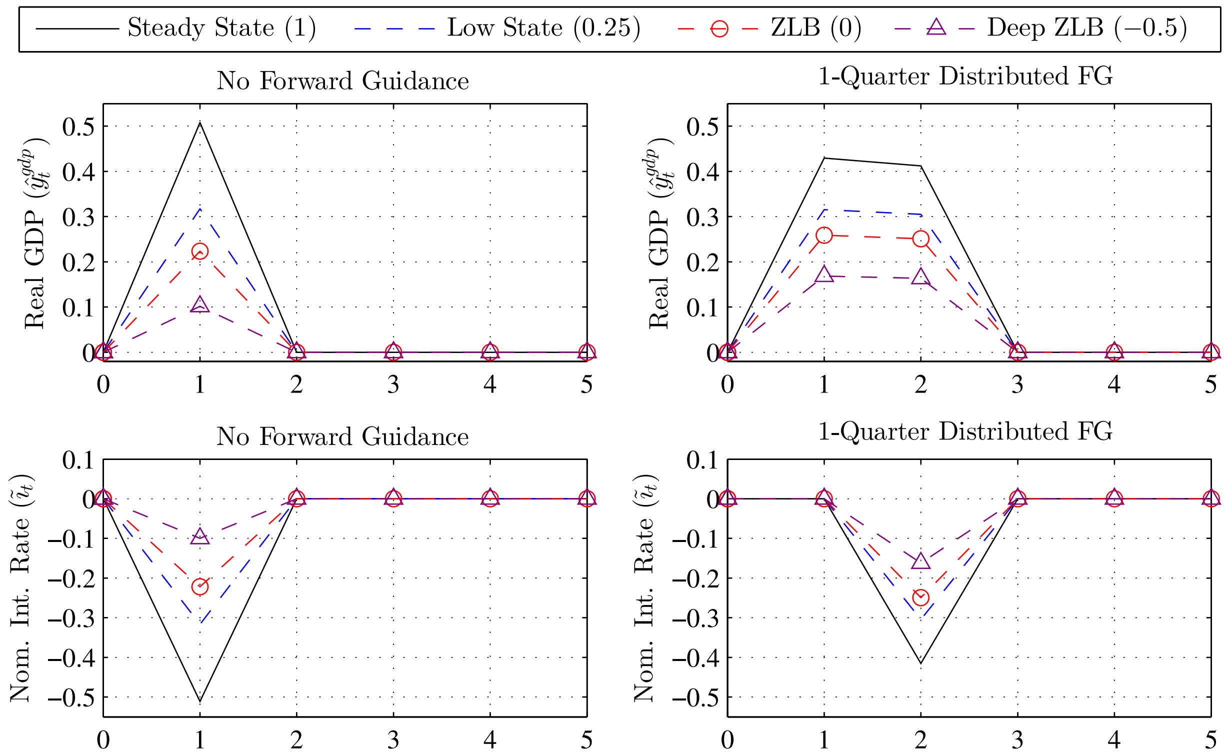


Figure 5: Generalized impulse responses to a -0.5% monetary policy shock. Two types of monetary policy are examined: No forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and 1-quarter distributed forward guidance (right panels). Each line represents a simulation initialized at a specific notional interest rate. In each case, the weights on the 1-quarter distributed forward guidance shock are set to eliminate any feedback effects on the nominal interest rate.

less stimulative as the initial notional interest rate declines. In steady state ($\tilde{\imath}_0^* = 1$), a -0.5% shock (unanticipated or anticipated) generates the largest decline in the nominal interest rate and has the greatest stimulative effect on real GDP because the policy rate rarely falls by enough to hit its ZLB. The same shock has a smaller effect on real GDP when $\tilde{\imath}_0^* = 0.25$ because the current and expected nominal interest rates are closer to zero and, as a result, have less room to fall after the shock. The effect is further reduced when $\tilde{\imath}_0^*$ equals 0% and -0.5% since policy is even more constrained.

Two, an unanticipated shock is more stimulative on impact than a news shock when the economy is at steady state, while a news shock becomes relatively more stimulative as the policy rate approaches its ZLB. At steady state ($\tilde{\imath}_0^* = 1$), a -0.5% unanticipated shock initially increases real GDP by 0.51%, whereas a 1-quarter distributed forward guidance shock pushes up real GDP by 0.43%. That same shock raises real GDP by only 0.10% in a severe recession ($\tilde{\imath}_0^* = -0.5$), while the distributed shock increases real GDP by 0.17%. The relative effectiveness of unanticipated shocks versus news shocks depends on how far the current and expected nominal interest rates are from the ZLB. When $\tilde{\imath}_0^* = 1$, the initial notional rate is high enough that the ZLB binds only 1% of the time. The low probability enables the entire unanticipated shock to stimulate the economy most of the time. That result changes when $\tilde{\imath}_0^* = -0.5$. At that state, the ZLB binds 67% of the time, so unanticipated shocks hardly have any effect. The stimulative effect of the distributed shock also declines as the policy rate approaches zero. Its economic effects, however, depend on how close the expected nominal rate, as opposed to the current nominal rate, is to the ZLB. Therefore, if the economy is expected to improve, then the expected nominal rate will be higher than the current rate, which gives news shocks a larger margin to stimulate the economy than unanticipated shocks.

A key policy implication of our results is that forward guidance is more stimulative when used proactively (i.e., at the onset of a recession). In 2008, the Fed only used forward guidance after the rate fell to its ZLB. The Fed could have also tried to coordinate conventional monetary policy and forward guidance. Our form of distributed forward guidance is a type of policy coordination, where the central bank provides conventional policy shocks to offset the stimulus from forward guidance. Alternatively, the central bank could redistribute more of the weight from α_1 to α_0 to lower the current nominal rate and the expected future rate. Any benefit from that policy, however, would be small, and possibly negative, since there would be less weight on the news shock, which provides more stimulus than the contemporaneous shock when the ZLB is more likely to bind.

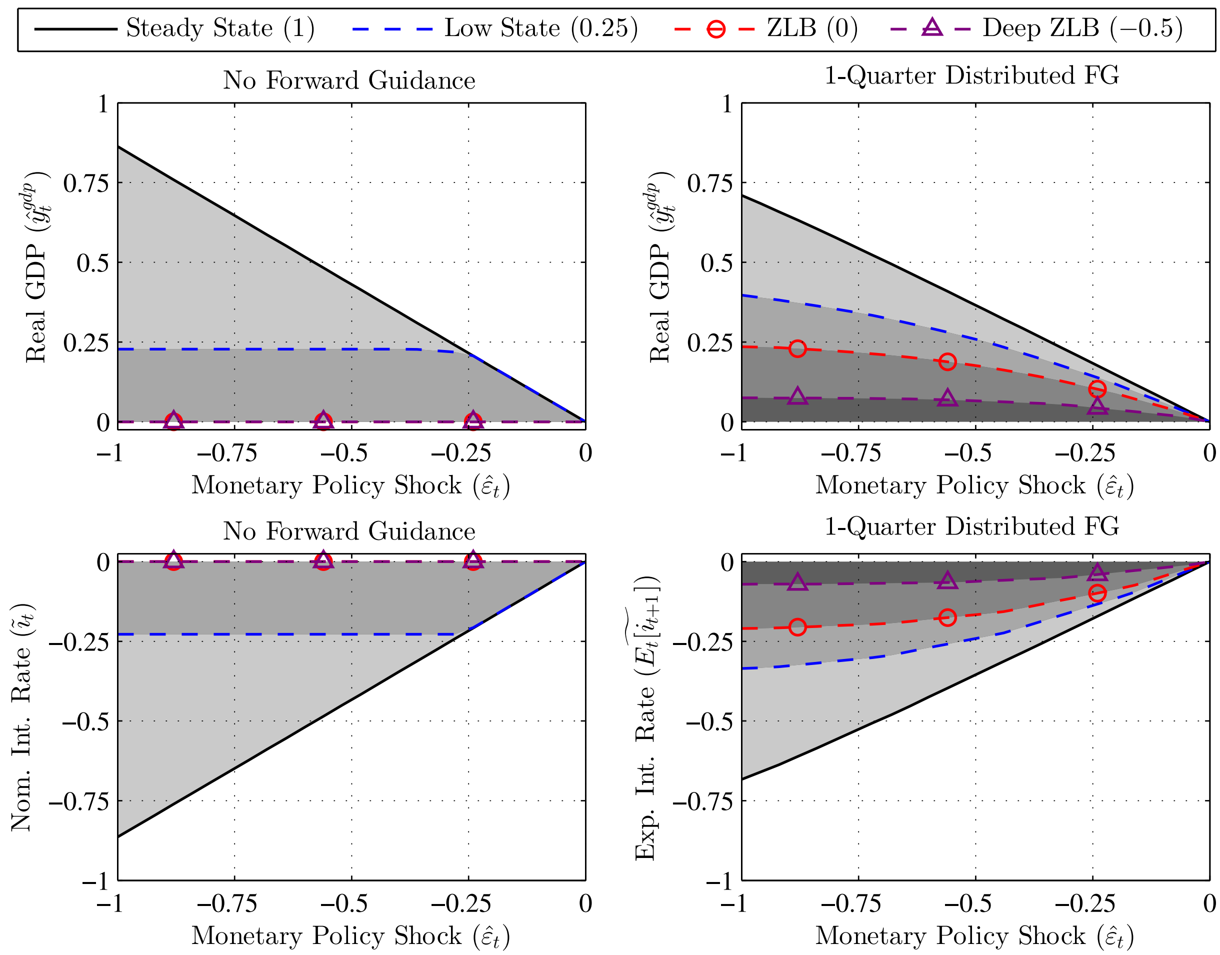


Figure 6: Decision rules with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and distributed forward guidance (right panels). Each line represents a cross section of the decision rules. In each cross section, the weights on the distributed forward guidance shock (α_0 and α_1) are set to eliminate any feedback effects on the nominal interest rate.

4.4 Size of the Shock The size of the monetary policy shock is another factor that determines whether an unanticipated or distributed news shock is more stimulative on impact. Figure 6 plots the decision rules as a function of the entire distribution of policy shocks with no forward guidance (left panels) and 1-quarter distributed forward guidance (right panels) for the same four initial notional interest rates examined in figure 5. In each cross section, the distributed forward guidance weights (α_0 and α_1) are set so the news shock has no feedback effects on the nominal interest rate.

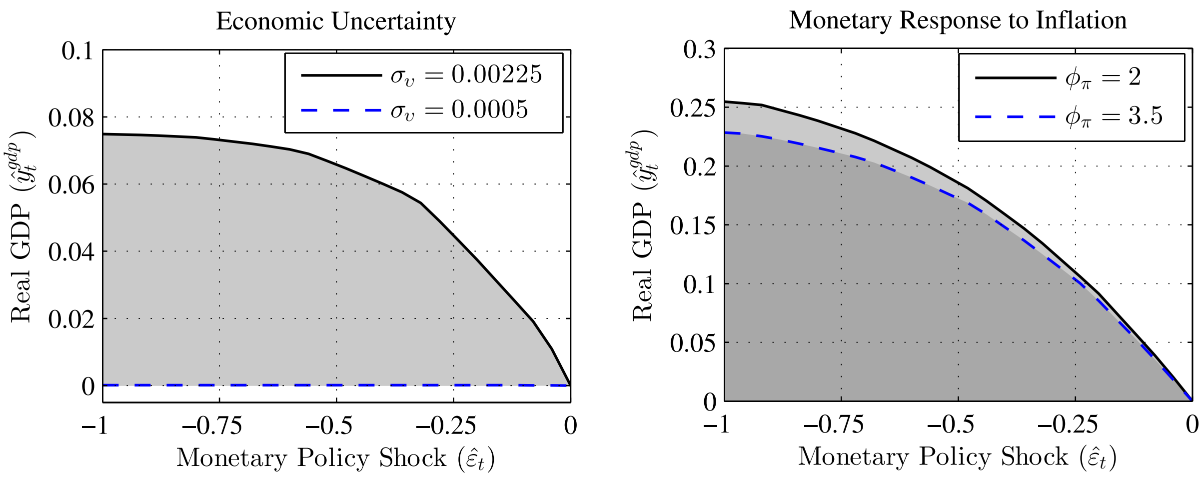
A comparison of the right and left panels of figure 6 enables us to determine whether an unanticipated shock or news shock is more stimulative in each state without having the analysis distorted by the feedback effect on the current nominal interest rate. When the economy is at steady state ($\tilde{\imath}_t^* = 1$), an unanticipated shock (solid line, left panel) always raises real GDP more on impact than a 1-quarter distributed forward guidance shock (solid line, right panel). The economic effects of an unanticipated shock, however, are more limited when the initial notional interest rate is low enough that the shock causes the ZLB to bind. If the economy is expected to improve, situations exist in which a promise to lower future nominal interest rates generates a larger increase in real GDP than an equivalent shock to the current nominal rate, which cannot fall below the ZLB. Consider the case where $\tilde{\imath}_t^* = 0.25$. A small unanticipated shock, $\hat{\varepsilon}_t > -0.26$, does not drive the nominal interest rate to its ZLB, so the jump in real GDP is higher than with a 1-quarter distributed shock. A moderate-sized unanticipated shock, $-0.42 < \hat{\varepsilon}_t < -0.26$, reduces the nominal interest rate to zero, but the initial stimulative effect is still stronger than the effect of distributed forward guidance. A large unanticipated shock, $\hat{\varepsilon}_t < -0.42$, causes an increasingly smaller rise in real GDP than the same distributed news shock. The upshot is that any forward guidance communicated when the policy rate is close to zero can generate a larger boost in real GDP than conventional open market operations as long as the news produces a meaningful revision in expected future interest rates.

When a recession is severe enough to cause the ZLB to bind ($\tilde{\imath}_t^* = 0$), distributed forward guidance is always more stimulative because an unanticipated shock cannot reduce the nominal interest rate. In a deeper recession ($\tilde{\imath}_t^* = -0.5$), the probability of exiting the ZLB next period becomes smaller, which reduces the expected nominal rate and limits the stimulative effect of forward guidance. In fact, it is possible that forward guidance will not have any stimulative effect if the initial notional rate is sufficiently low. These results reinforce our finding from figure 5 that the stimulative effect of forward guidance is much more limited in a severely depressed economy, which provides further support for communicating forward guidance early in an economic downturn.

	Initial Notional Interest Rate			
Uncertainty	Steady State (1)	Low State (0.25)	ZLB (0)	Deep ZLB (-0.5)
High ($\sigma_\upsilon = 0.00225$)	0.43	0.32	0.26	0.17
Low ($\sigma_\upsilon = 0.0005$)	0.44	0.41	0.26	0.09

Table 2: Impact effect on real GDP in response to a -0.5% 1-quarter distributed forward guidance shock.

4.5 MONETARY POLICY AND ECONOMIC UNCERTAINTY The degree of economic uncertainty and the expected stance of monetary policy when the ZLB does not bind also influence the effectiveness of forward guidance. Table 2 shows the impact effect on real GDP from a -0.5% 1-quarter distributed forward guidance shock under high and low levels of uncertainty about the future path of the discount factor. The high calibration represents the degree of uncertainty in our baseline model, while the low calibration approximates the behavior of our model under perfect foresight. The consequences of economic uncertainty are state dependent. When the economy is in a deep recession ($\tilde{\imath}_0^* = -0.5$), higher uncertainty increases the stimulative effect of forward guidance, whereas the stimulative effect is smaller when the economy is in a low state ($\tilde{\imath}_0^* = 0.25$). In steady state ($\tilde{\imath}_0^* = 1$) and when the interest rate is right at the ZLB ($\tilde{\imath}_0^* = 0$), uncertainty has little effect.



(a) Decision rules with high uncertainty, $\sigma_v = 0.00225$, (solid line) and low uncertainty, $\sigma_v = 0.0005$, (dashed line). In this cross section of the decision rules, the initial notional rate equals -0.5%.

(b) Decision rules with a low inflation response, $\phi_\pi = 2$, (solid line) and a high inflation response, $\phi_\pi = 3.5$, (dashed line). In this cross section of the decision rules, the initial notional rate equals zero.

Figure 7: Effect of economic uncertainty (panel a) and the expected monetary response to inflation (panel b).

To further illustrate how economic uncertainty affects forward guidance, figure 7a plots the 1-quarter distributed forward guidance decision rules when the economy is in a deep recession. In this state, lower uncertainty about the discount factor makes households more confident that the nominal interest rate will remain at or near the ZLB. That is, positive discount factor shocks are less likely to warrant an increase in the policy rate. Therefore, the central bank has a smaller margin to reduce the expected interest rate, which limits the stimulative effect of forward guidance. Right at the ZLB, the degree of economic uncertainty has no effect on the probability of leaving the ZLB. When economic conditions warrant a low policy rate, less uncertainty causes the future nominal interest rate distribution to be less constrained, which generates a larger margin for the news to stimulate the economy. In steady state, the short-term probability of hitting the ZLB is low, so the degree of uncertainty has very little influence on the effectiveness of forward guidance.

When the Fed lowered its policy rate to zero in December 2008, there was a high degree of uncertainty about future economic conditions that persisted for several years. Our results suggest the Fed could have taken advantage of the high uncertainty by communicating its intention to keep the federal funds rate low for several years. For example, the FOMC did not use specific language about its future policy rate path until it began using date-based forward guidance in August 2011. By that time, however, forecasters had become much more pessimistic about future economic conditions. Thus, the date-based language likely would have been much more effective at boosting real GDP if it had been used in 2008 or 2009 when economic forecasts were far more uncertain.

Figure 7b shows a larger inflation coefficient in the monetary policy rule reduces the stimulative effect of forward guidance when the ZLB binds. In that state of the economy, inflation is well below its target rate. A larger ϕ_π implies inflation must rise more and be closer to its target for the policy rule to call for an increase in the interest rate above the ZLB. As a consequence, households expect lower future nominal interest rates, which reduces the margin for forward guidance to stimulate the economy. When communicating forward guidance, central banks may be tempted to affirm their commitment to fighting inflation to contain the inflationary pressures generated by that policy. Our results, however, suggest that such a statement would reduce the effectiveness of forward guidance.

4.6 SPEED OF THE RECOVERY Another important determinant of the stimulative effect of forward guidance is how quickly households expect the economy to recover from a recession where the ZLB binds. Unfortunately, the continuous process for the discount factor makes it impossible to change the probability of leaving the ZLB (i.e., the speed of the recovery) without simultaneously changing the probability of going to the ZLB (i.e., the likelihood of a recession). To avoid that problem, we assume the discount factor follows a 2-state Markov chain with transition matrix $\Pr\{s_t = j | s_{t-1} = i\} = p_{ij}$ for $i, j \in \{1, 2\}$. The discount factor is at its steady state in state 1, whereas the discount factor is high enough for the ZLB to bind in state 2. We set p_{12} equal to 1% and then conduct sensitivity analysis on p_{21}, which determines the expected speed of the recovery.

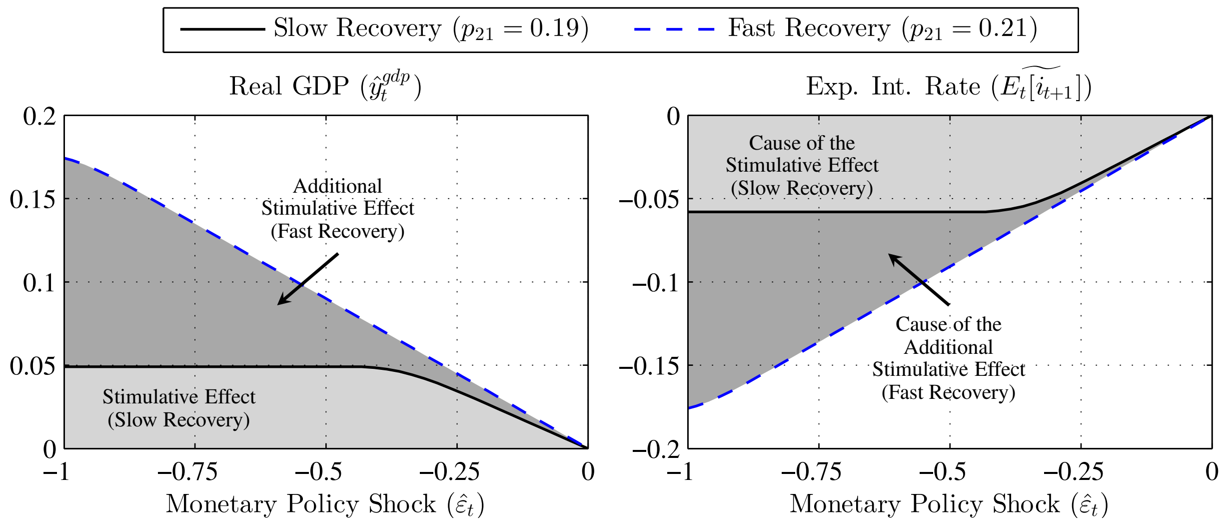


Figure 8: The stimulative effect of 1-quarter forward guidance, $(\alpha_0, \alpha_1) = (0, 1)$, given a slow recovery (solid line) and a fast recovery (dashed line). In this cross section of the decision rules, the initial notional interest rate equals zero.

Figure 8 shows decision rules with 1-quarter forward guidance as a function of the monetary policy shock given a relatively slow recovery ($p_{21} = 0.19$, solid line) and a 10% increase in the probably of exiting the ZLB ($p_{21} = 0.21$, dashed line). The light-shaded region represents the stimulative effect of forward guidance when the economy recovers slowly and the dark-shaded region is the marginal effect of the faster recovery. As in figure 1, the initial notional interest rate equals zero in this cross section of the decision rules. The values of the transition probabilities are chosen to highlight the qualitative difference in the stimulative effect of forward guidance.

The stimulative effect of forward guidance is dampened when households expect a slower economic recovery. A less rapid return to steady state reduces demand and lowers the expected nominal interest rate. The smaller jump in the expected nominal rate implies that a promise to maintain a low policy rate in the future will have a weaker effect on real GDP because there is a smaller margin for policy to push down the expected nominal rate in order to stimulate real GDP.[11]

The decision rules under the slow recovery exhibit a kink due to the lower expected nominal interest rate. For small news shocks, $\hat{\varepsilon}_t > -0.25\%$, the expected nominal rate decreases linearly since expectations are a convex combination of the future interest rates across the two states. For

[11]Levin et al. (2010) also show a slower expected recovery hinders forward guidance. They assume a real rate shock hits the economy, decays at a constant rate for four periods, and then switches to a slower rate of decay. Eggertsson and Mehrotra (2014) argue that forward guidance is less effective when the economy is in a near-permanent slump.

large news shocks, $\hat{\varepsilon}_t \leq -0.25\%$, the expected nominal rate is at the ZLB in both states, so its decision rule is flat. With a fast recovery, however, large news shocks do not push the expected nominal rate to its ZLB, so they generate a larger increase in real GDP that grows with the size of the news shock. While the marginal effect depends on the transition probability and the point at which the ZLB binds in expectation, these results demonstrate that forward guidance has a more limited stimulative effect if the policy causes households to revise their expectations about the economy or is communicated at the same time households learn about a weaker economic outlook.

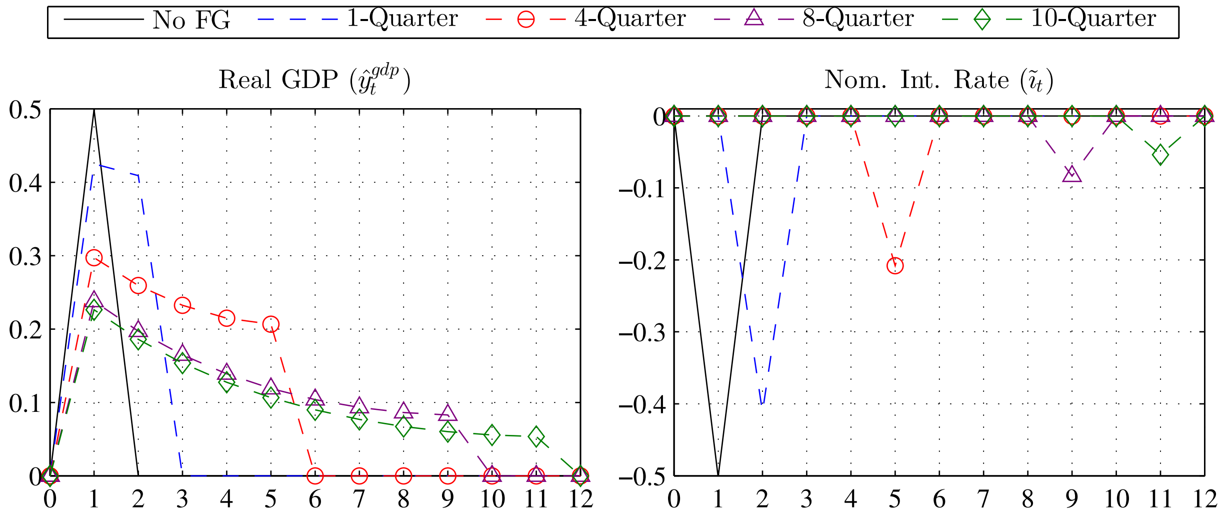
4.7 SUMMARY The central bank can stimulate economic activity by either using conventional monetary policy to lower the current nominal interest rate or forward guidance to lower the expected future nominal interest rate. The stimulative effect of each policy depends on how much conventional policy and forward guidance can lower the current nominal rate and expected future nominal rate, respectively. When nominal rates are sufficiently above zero, a conventional monetary policy shock has a larger stimulative effect because households value cuts to the current nominal rate more than to the expected nominal rate. As the current nominal rate approaches the ZLB, the expected nominal rate is usually higher than the current nominal rate. The central bank then has more room to lower the future nominal rate, so forward guidance usually has a stronger impact on economic activity. In a ZLB or near ZLB environment, the stimulative effect of forward guidance increases as the expected future nominal rate rises. For example, a stronger initial state of the economy, a less aggressive stance of monetary policy away from the ZLB, and a faster economic recovery all push up the expected future nominal rate, which causes forward guidance to be more effective. High levels of economic uncertainty generate the largest margin to reduce the expected future nominal rate when the current nominal rate is in a deep ZLB state, whereas lower uncertainty produces a higher expected rate when the current nominal rate is slightly above zero.

5 LONGER HORIZON RESULTS

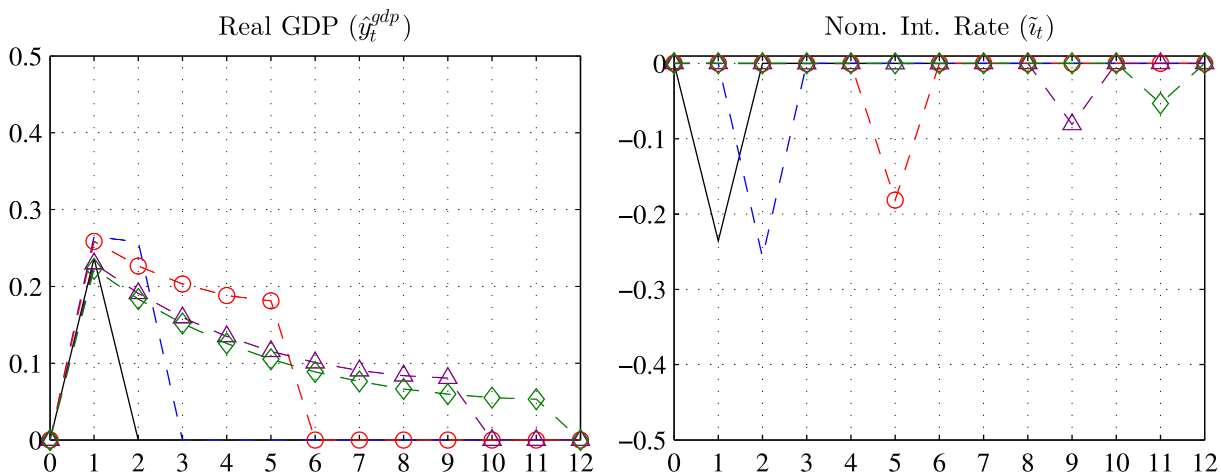
This section first examines the stimulative effects of forward guidance over horizons up to 10 quarters. We then show how a simultaneous demand shock obscures the impact of forward guidance. It concludes by comparing our approach to modeling forward guidance to an interest rate peg.

5.1 METHODOLOGY Our results in section 4 use Gauss-Hermite quadrature to evaluate expectations. That approach allows us to obtain an accurate approximation of the decision rules and to quantify the stimulative effect of forward guidance for many different monetary policy shocks, which is important because the responses of key economic variables are nonlinear functions of the shock size. Using that technique, appendix A presents the economic effects of 2-quarter forward guidance across all policy shocks. That solution method, however, is numerically infeasible with longer forward guidance horizons because the state space grows exponentially with the horizon.

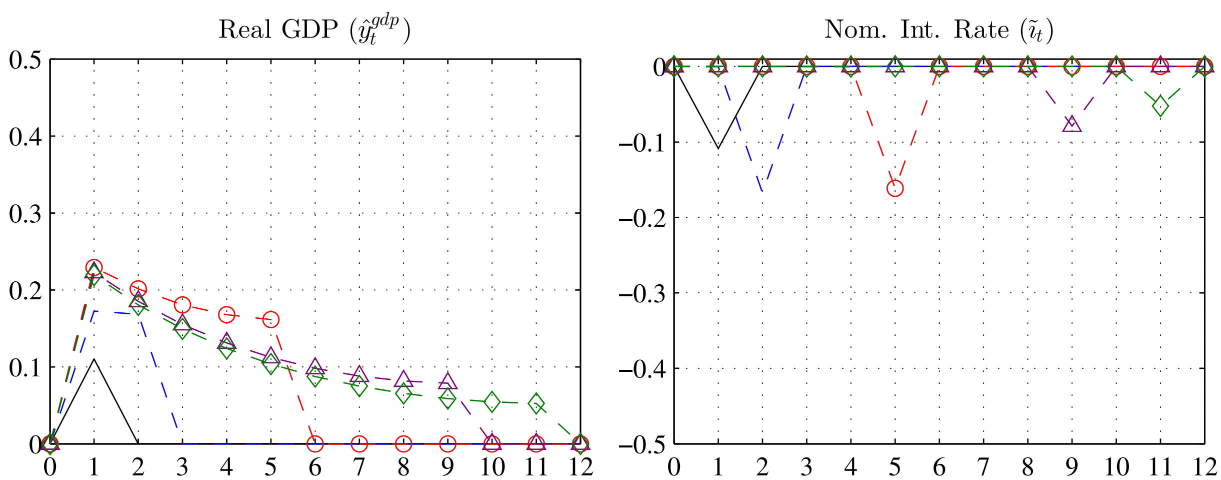
We reduce the dimensionality of the state space when analyzing horizons beyond 2 quarters by discretizing the news process using the method in Tauchen (1986). Specifically, we assign three values for each monetary policy shock, $(-0.6, 0, 0.6)$, and then calculate the probabilities of the transitional events. Tauchen's (1986) method is particularly useful for examining longer forward guidance horizons because it enables us to analyze the effects of specific shocks to the news process without having to solve the model for several other possible realizations of the shocks. See appendix E for more details on how this solution procedure differs from the previous method.



(a) Simulations initialized at steady state.



(b) Simulations initialized at a notional interest rate equal to zero.



(c) Simulations initialized at a notional interest rate equal to -0.5%.

Figure 9: Generalized impulse responses to a -0.6% monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, and distributed forward guidance at various states of the economy. In each simulation, the weights on the distributed forward guidance shock (α_j, $j = 0, 1, \ldots, q$) are set to eliminate any feedback effects on the policy rate.

5.2 FORWARD GUIDANCE HORIZON Figure 9 shows generalized impulse responses to a -0.6% monetary policy shock distributed over 1-, 4-, 8-, and 10-quarter forward guidance horizons. For each horizon, the weights on the shocks are selected to eliminate any feedback effects on the policy rate. We focus on distributed forward guidance because it better reflects actual policy and allows us to obtain an accurate comparison across the various horizons. In the top, middle, and bottom panels, the simulations are initialized at steady state ($\tilde{\imath}_0^* = 1$), the ZLB ($\tilde{\imath}_0^* = 0$), and a severe recession ($\tilde{\imath}_0^* = -0.5$). The weights corresponding to each initial state are shown in appendix F.

When the economy is initialized at steady state ($\tilde{\imath}_0^* = 1$, top row), the unanticipated monetary policy shock raises real GDP more on impact than the distributed news shock, regardless of the forward guidance horizon. Unlike the effects of an unanticipated shock, which disappear after period 1, the impact of a q-quarter distributed forward guidance shock persists for q more quarters. To prevent the policy rate from changing over the horizon, any future deviation from the Taylor rule at the end of the horizon necessitates a deviation from the rule over the whole horizon. Therefore, distributed forward guidance shifts some of the weight on the policy shock from period $q + 1$ to periods 1 to q to eliminate the feedback effect on the nominal interest rate. The result is the size of the shock in period $q + 1$ becomes smaller as q increases (i.e., α_q declines as q rises). The smaller shock dampens the initial rise in real GDP, but the increase persists over the entire forward guidance horizon. Beyond period $q + 1$, the news shocks do not have any effect on the economy.[12]

When the economy begins in a recession that is just severe enough for the ZLB to bind ($\tilde{\imath}_0^* = 0$, middle row), the initial rise in real GDP is similar across all forward guidance horizons. The boost in real GDP, however, is smaller in every period over the forward guidance horizon than occurs when the economy is initialized at steady state. The reduced stimulative effect is due to the smaller margin that the central bank has to lower expected nominal interest rates over the next few periods. In an economic downturn similar to the Great Recession ($\tilde{\imath}_0^* = -0.5$, bottom row), the stimulative effect of forward guidance is even more limited, especially over short horizons. At longer horizons, the response of real GDP in every quarter is mostly unaffected by the initial state of the economy.

There are two key takeaways from our results. One, longer forward guidance horizons spread the effect of the news across the entire horizon, instead of generating increasingly larger impact effects on real GDP. Two, poorer economic conditions limit the stimulative effect of forward guidance in the short run, but those negative effects have a much smaller impact over longer horizons. These findings are in sharp contrast to Del Negro et al. (2015) and McKay et al. (2016), who show that textbook New Keynesian models lead to increasingly large stimulative effects of forward guidance. Our exercises differ in that we incorporate the ZLB constraint into households' expectations and fix the total amount of news when comparing the various forward guidance horizons.

To quantify the cumulative effect of the forward guidance policies shown in figure 9 across the entire horizon, we calculate the present value of the percent change in real GDP in every period:

$$\text{Cumulative Effect } \hat{y}(q) = \frac{1}{N}\sum_{j=1}^{N}\sum_{t=1}^{q+1}\frac{100(y_{j,t}^{\varepsilon}/y_{j,t}^{no\ \varepsilon}-1)}{\prod_{k=2}^{t} r_{j,k}},$$

where $y_{j,t}^{no\ \varepsilon}$ is real GDP conditional on draw j of the shocks, $y_{j,t}^{\varepsilon}$ is real GDP conditional on the same draw of shocks, except $\hat{\varepsilon}_1 = -0.6\%$, $r_{j,t}$ is the gross real interest rate from draw j, and N

[12] De Graeve et al. (2014) show that if the model contains backward-looking endogenous state variables, such as habit formation or inflation indexation, then the effects of the policy will persist beyond the forward guidance horizon.

is the number of simulations. Table 3 shows the present value of the cumulative percent change in real GDP over various forward guidance horizons in response to a -0.6% monetary policy shock.

	Forward Guidance Horizon				
Initial State of the Economy	0	1	4	8	10
Steady State ($\tilde{\imath}_0^* = 1$)	0.50	0.83	1.19	1.20	1.17
Recession ($\tilde{\imath}_0^* = 0$)	0.23	0.51	1.00	1.09	1.09
Deep Recession ($\tilde{\imath}_0^* = -0.5$)	0.11	0.33	0.87	1.03	1.04

Table 3: Present value of the cumulative percent change in real GDP in response to a -0.6% monetary policy shock.

For all states of the economy, q-quarter distributed forward guidance always has a larger cumulative effect on real GDP than an unanticipated shock. The size of the cumulative effect, however, depends on both the state of the economy and the forward guidance horizon. In steady state ($\tilde{\imath}_0^* = 1$), extending the forward guidance horizon to 4 quarters increases the cumulative effect on real GDP, but provides little effect thereafter. At the ZLB ($\tilde{\imath}_0^* = 0$), increasing the horizon from 4 to 8 quarters only raises the present value of real GDP by 0.09%, while increasing the horizon beyond 8 quarters has no additional effect. In a deep recession ($\tilde{\imath}_0^* = -0.5$), an increase in the horizon from 4 to 8 quarters boosts the present value of real GDP by 0.16% but has no meaningful impact beyond 8 quarters. Those results indicate it is more beneficial to extend the forward guidance horizon when the economy is facing worse economic conditions. The hump-shaped pattern over the forward guidance horizon, however, indicates that the central bank faces limits on how far forward guidance can extend into the future and continue to add economic stimulus. That finding suggests there is an optimal forward guidance horizon, which depends on the state of the economy.

Carlstrom et al. (2015) and De Graeve et al. (2014) show that endogenous state variables can affect the dynamics generated by forward guidance. To test the robustness of our results, appendix B extends our model in section 3 to include habit formation. That feature dampens, delays, and extends the stimulative effect of forward guidance, but all of our key findings continue to hold. We separately examined inflation indexation, but that feature had a much smaller quantitative effect.

5.3 FORWARD GUIDANCE AND LOWER DEMAND

Despite the Fed's use of forward guidance and other unconventional policy measures since late 2008, professional forecasts of real GDP remained low and some even fell in response to recent FOMC statements. One plausible explanation for the weak real GDP forecasts is the forward guidance announcements were accompanied by weak economic assessments by the Fed. Using simulations, this section reconciles the apparent contradiction between the effects of news shocks in our model and forecasts observed in the data.

Figure 10 compares the economic effects of a decline in demand and an announcement of 4-quarter distributed forward guidance. To assess their combined effects, we compute generalized impulse responses to a simultaneous 1 standard deviation positive discount factor shock that reduces demand and a -0.6% forward guidance shock distributed over 4 quarters (solid line). Those responses are then compared to the responses with only the forward guidance shock (dashed line) and the responses with only the discount factor shock (dash-dotted line). The simulations are initialized at a notional interest rate equal to -0.5%. The distance between the dashed line and the solid line measures the effect of the negative demand shock, whereas the distance between the dash-dotted line and the solid line is the marginal benefit of 4-quarter distributed forward guid-

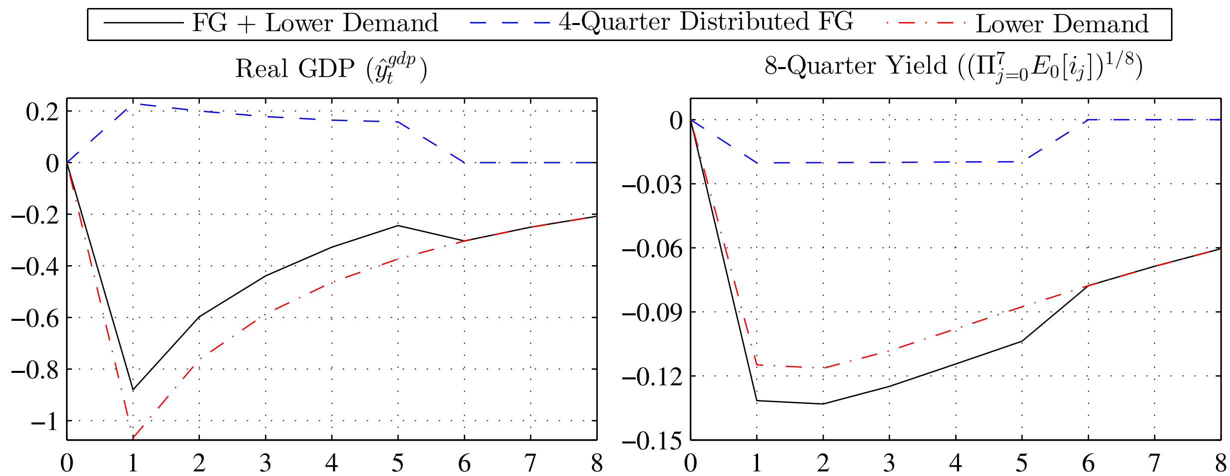


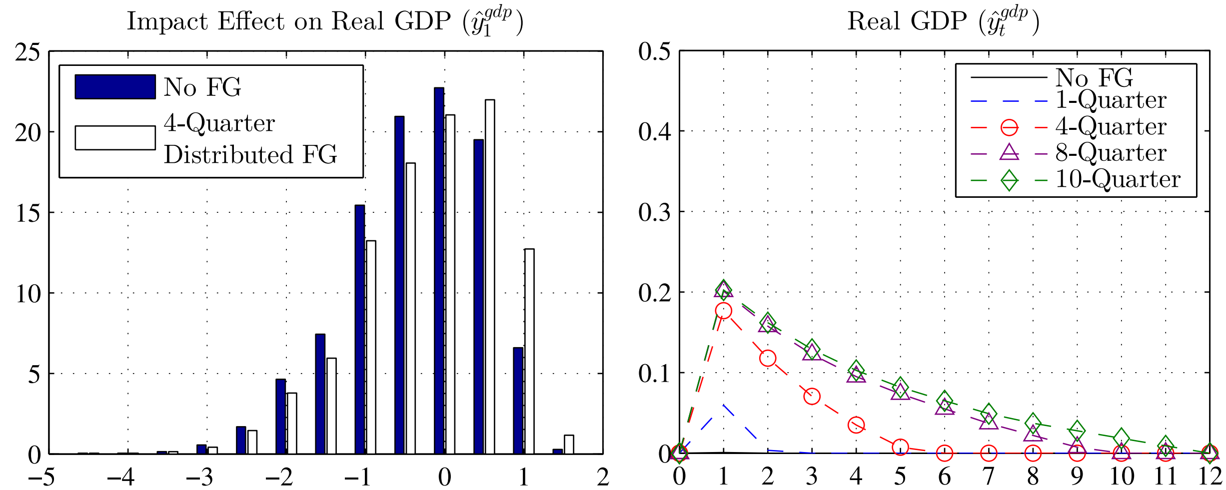
Figure 10: Generalized impulse responses to a 1 standard deviation positive discount factor shock and a -0.6% monetary policy shock with 4-quarter distributed forward guidance (solid line). The combined responses are compared to responses with only the monetary policy shock (dashed line) and only the discount factor shock (dash-dotted line). Each simulation is initialized at a notional rate equal to -0.5% to reflect the environment of recent forward guidance.

ance. Since the simulations are consistent with households' expectations, we interpret the GIRFs as consensus forecasts made by households in period zero. In figure 10, the forecasts incorporate forward guidance about a future policy rate cut and/or an expected increase in the discount factor.

An announcement in period 1 of 4-quarter distributed forward guidance reduces the 8-quarter yield and raises the real GDP forecast, whereas a forecast of a negative demand shock in period 1 also pushes down the yield curve but at the expense of lower expected real GDP. When the two shocks simultaneously hit the economy, the yield curve shifts down, and the forecast for the path of real GDP depends on which of the two shocks dominate. In figure 10, the discount factor shock dominates the forward guidance announcement, so the real GDP forecast falls. Those findings illustrate two important points. One, identifying the source of empirically-observed changes in interest rate forecasts is challenging because households often receive forward guidance and information about current and future economic conditions at the same time. Two, forward guidance is stimulative in the absence of any other shocks, but the observed effect on real GDP forecasts is smaller or even negative if another shock is expected to simultaneously reduce current demand.

In figure 10, households forecast a specific discount factor shock, but we can also simulate the model over a selected range of shocks. That approach is useful for analyzing forward guidance in the aftermath of the Great Recession because it was when the economy recovered slower than households expected. Specifically, we restrict our sample of shocks to values of the discount factor that keep the policy rate at its ZLB in the absence of forward guidance. That set of shocks is used to generate a distribution of real GDP outcomes with and without forward guidance. The differences between those two distributions indicate the effectiveness of recent forward guidance.

Figure 11a shows the distribution of the forecasted impact effect on real GDP from generalized impulse responses with no forward guidance (dark bars) and a -0.6% 4-quarter distributed forward guidance shock (light bars). The simulations used to produce both distributions are initialized at the deep ZLB state ($\tilde{\imath}_0^* = -0.5$) and are based on sequences of discount factor shocks that keep the nominal interest rate at zero for a minimum of five periods, so the economy does not recover as fast as households expect. Those expectations, however, are strong enough to provide the central bank



(a) Distribution of the impact effect on real GDP.

(b) Mean path of real GDP across various horizons.

Figure 11: Generalized impulse responses to a -0.6% policy shock when the economy recovers slower than expected. Each simulation is initialized at a notional rate equal to -0.5% and the policy rate remains at zero for 11 quarters.

with a small margin to lower expected nominal interest rates even though the actual nominal rate remains at its ZLB over the entire forward guidance horizon. A comparison of both distributions for real GDP reveals that forward guidance shifts the distribution to the right, so real GDP falls less often on impact. That is, the impact effect on real GDP is negative in 62% of the simulations without forward guidance and drops to 53% with forward guidance. In both cases, the declines in real GDP are caused by a negative demand shock, but the forward guidance shock is strong enough in some cases to prevent real GDP from falling. In the remaining portion of the distribution where real GDP declines with forward guidance, the demand shocks are large enough to mask the stimulative effect of the news shock. Those findings reinforce our contention that forward guidance boosts real GDP, even though the evidence from recent forward guidance might suggest otherwise.

We also use this technique to regenerate the generalized impulse responses in figure 9c based on sequences of discount factor shocks that keep the policy rate at zero for the next 11 quarters, so they are comparable to the results with an interest rate peg. Figure 11b shows forward guidance continues to stimulate output because households expect the policy rate to rise. As in figure 9c, lengthening the forward guidance horizon increases the cumulative effect of real GDP to a certain point, even though the policy rate remains at zero for the entire horizon. The cumulative effect on real GDP, however, is only about half as large as when the economy recovers at the expected rate. A key takeaway is that the effectiveness of forward guidance is overstated whenever the analysis does not account for changes in demand that occur at the same time the policy is communicated.

We do not take a position on why demand shifts in our model. In reality, there are several reasons why the discount factor may change when forward guidance is announced. One, households may interpret news of lower future policy rates as a signal of a weaker economy or a slower economic recovery. Two, policy statements may also provide a forecast of economic conditions that is worse than private forecasts, which leads households to revise their forecasts downward. Three, other sources may provide information that the economy is not performing as well as previously expected at the same time as the forward guidance announcement. Any of those scenarios could decrease real GDP, even if forward guidance is successful at reducing expected policy rates.

5.4 INTEREST RATE PEG Modeling forward guidance with an interest rate peg is a special case of our news shock approach. This section uses the same model as in section 3, but modifies (4) with a Markov process that governs whether the policy rate is pegged. The monetary policy rule is

$$i_t = \begin{cases} \max\{\underline{\imath}, i_t^*\} & \text{for } e_t = 0 \\ \underline{\imath} & \text{for } e_t = 1 \end{cases}.$$

In period t, the nominal interest rate is determined endogenously when $e_t = 0$ and is exogenously pegged at its ZLB when $e_t = 1$. Forward guidance is characterized by a vector of interest rate policies, $[e_t, e_{t+1}, \ldots, e_{t+q}]$, communicated to households in period t over horizon q. Our approach to modeling an interest rate peg is unique because it allows the ZLB to bind without any forward guidance.[13] In other words, when the central bank does *not* peg the policy rate, households still form expectations about the possibility that economic conditions could cause the ZLB to bind.

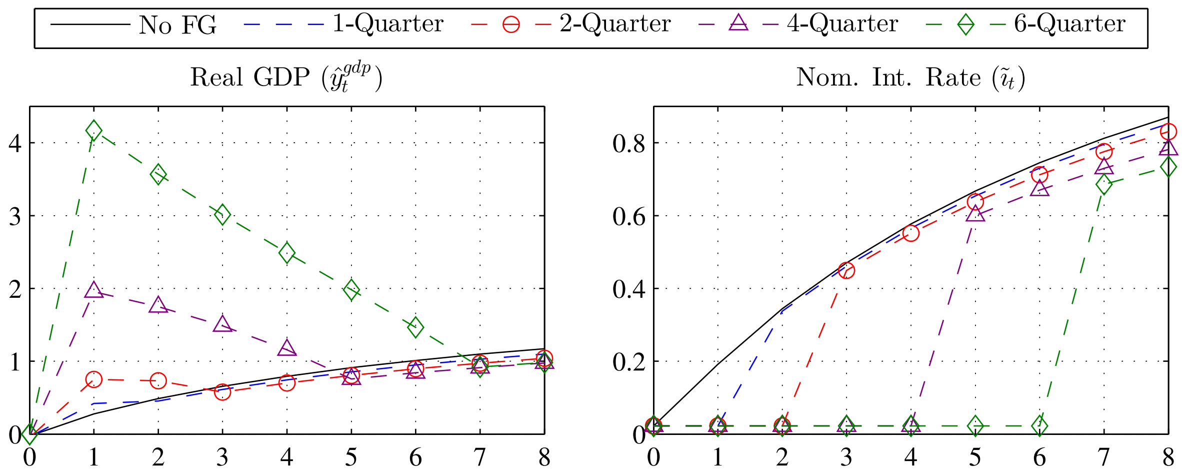


Figure 12: Economic effects of alternative interest rate pegs when the initial notional interest rate equals zero.

Figure 12 shows the effects of various interest rate pegs when the initial notional rate equals zero. With a 1-quarter peg, forward guidance begins with $[e_0, e_1] = [1, 1]$, where the central bank promises to keep the nominal interest rate at zero until next period. Forward guidance then transitions to $[e_1, e_2] = [1, 0]$ in the first period, where the central bank holds the current nominal rate at its ZLB but allows the peg to lapse in period 2. Without a peg ($[e_t, e_{t+1}] = [0, 0]$) in period 2 and beyond, households expect the nominal rate to rise as the economy recovers. With a q-quarter peg, forward guidance begins with $[e_0, \ldots, e_q] = [1, \ldots, 1]$, which guarantees the nominal rate will remain at zero for q periods. Forward guidance then transitions to states that reflect the number of periods remaining in the peg. See appendix F for details on how we compute the interest rate peg.

With an interest rate peg, longer forward guidance horizons generate increasingly larger impact effects on real GDP because every additional quarter the policy rate is pegged at zero is equivalent to a news shock that is large enough to drive the expected nominal rate to its ZLB. Therefore, an interest rate peg gives the central bank a much stronger ability to affect expected nominal rates than is observed in the data. The peg also ignores the possibility the policy rate could rise before or after the target date, which is inconsistent with the threshold-based forward guidance used from

[13] Blake (2012) examines alternate ways to peg the policy rate in a model with an endogenous monetary policy rule.

December 2012-January 2014 and the state-contingent nature of earlier statements. In other words, the peg implies there is no uncertainty about future interest rates, which is unlikely and at odds with recent data. After all, if households knew the policy rate would remain at zero, no one would spend time looking at job reports and other data to try to figure out when the central bank will raise rates.

Instead of pegging the nominal rate at its ZLB, it is possible to peg a specific path for the interest rate, so it is lower than the path that would occur without any forward guidance. That specification would be closer to our news shock approach, but it would still create a degenerate distribution for the future nominal interest rate because the peg would not depend on future economic conditions.

6 Case Studies of Federal Reserve Forward Guidance

This section uses the qualitative predictions of our theoretical model to help explain the economic effects of three recent FOMC policy statements that communicated date-based forward guidance.

6.1 2011 Policy Statement On August 9, 2011, the FOMC announced it "anticipates that economic conditions. . . are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013," which was the Committee's first use of date-based forward guidance. It also said, "The Committee now expects a somewhat slower pace of recovery over coming quarters," but the Fed's quantitative easing policy was unchanged, which makes this statement ideal to study.

Blue Chip forecasts of interest rates and real GDP changed after the August 9th FOMC statement was released. Assessing the effect of that statement on economic forecasts is complicated by a downward revision of GDP on July 29, 2011, which reduced real GDP growth in most quarters since the Great Recession. For example, real GDP growth in 2011Q1 (2008Q4) declined from 1.9% (7.0%) to 0.4% (9.2%). To separate the impact of the two events, we follow Crump et al. (2013) and use forecasts from the July and August 2011 Blue Chip Financial Forecasts (BCFF) survey and similar forecasts from the August 2011 Blue Chip Economic Indicators (BCEI) survey.

Date	2011Q4	2012Q1	2012Q2	2012Q3	2012Q4
BCFF (7/20-21)	0.14	0.26	0.43	0.75	1.08
BCEI (8/4-5)	0.13	0.19	0.29	0.50	0.77
BCFF (8/24-25)	0.07	0.09	0.12	0.14	0.20
Total Change	−0.07	−0.17	−0.31	−0.61	−0.88
Change following GDP	−0.01	−0.07	−0.13	−0.25	−0.31
Change following FOMC	−0.06	−0.10	−0.18	−0.36	−0.57

Table 4: Blue Chip consensus forecasts of the 3-month T-Bill rate. All values are annualized net rates.

Table 4 shows the consensus BCFF and BCEI forecasts of the 3-month T-bill rate from 2011Q4 to 2012Q4. The BCFF forecasts were made on July 20-21 before the GDP revision was released on July 29th, while the BCEI forecasts were made on August 4-5 just prior to the August 9th FOMC statement. The difference between the late-July and early-August forecasts is an implicit measure of the impact that the GDP revision had on the forecasts. The next BCFF forecasts were made on August 24-25. The difference between the BCEI's August 4-5 forecasts and the BCFF's August 24-25 forecasts indirectly measures the effect of the FOMC statement, which communicated date-based forward guidance and provided an assessment of current and expected economic conditions.

Data indicate the July 29th GDP revision led to a 13 basis point decline in the consensus forecast of the 2012Q2 3-month T-bill rate and a 31 basis point decline in the 2012Q4 rate. After the FOMC statement, there were even larger decreases in expected interest rates, with the 2012Q2 and 2012Q4 rates falling by an additional 18 and 57 basis points, respectively. In fact, 3-month T-bill forecasts for all of 2012 declined more after the FOMC statement than after the GDP revision. Following both events, the 2012Q4 rate was only 13 basis points higher than the 2011Q4 rate, which means forecasters believed the policy rate was unlikely to rise until 2013. Moessner (2013) and Raskin (2013) both find the FOMC statement had similar effects on interest rate forecasts.

While many forecasters expected the federal funds rate to remain near zero throughout 2012, some forecasters believed rates would rise in 2012, despite the FOMC's forward guidance in August 2011. For example, the average of the top 10% of the 8/24-25 BCFF forecasts for 2012Q4 was 0.54% and the highest forecast was 1.17%. A surprising 20% of the FOMC members also thought the federal funds rate would rise in 2012. Using options data, Swanson and Williams (2014) report that there was a 15% chance in late 2011 that the federal funds rate would rise above 0.5% by the end of 2012. Our news shock approach to modeling forward guidance accounts for the tails of the interest rate distribution, whereas there is no expectation of higher rates during an interest rate peg.

Date	2011Q4	2012Q1	2012Q2	2012Q3	2012Q4
BCFF (7/20-21)	3.09	2.75	2.97	3.07	3.17
BCEI (8/4-5)	2.53	2.38	2.59	2.81	2.88
BCFF (8/24-25)	2.17	2.13	2.44	2.69	2.90
Total Change	-0.92	-0.62	-0.53	-0.38	-0.27
Change following GDP	-0.56	-0.37	-0.38	-0.26	-0.29
Change following FOMC	-0.36	-0.25	-0.15	-0.12	0.02

Table 5: Blue Chip consensus forecasts of quarter-over-quarter real GDP growth. All values are annualized net rates.

Table 5 displays consensus forecasts of real GDP growth from 2011Q4 to 2012Q4. The forecast for 2011Q4 dropped 0.56 percentage points after the GDP revision but only 0.36 percentage points after the FOMC statement. The 2012Q4 forecast of real GDP growth declined by almost 0.3 percentage points after the GDP revision but slightly increased after the FOMC statement. A comparison of all forecast horizons through 2012 reveals that the decline in the forecasts of real GDP after the GDP revision is larger than the change that was observed after the FOMC statement.

Our model predicts forward guidance will reduce expected interest rates and push up real GDP when it is communicated without conflicting information. Data following the FOMC statement, however, indicate that near-term real GDP forecasts declined. Our theory provides two potential explanations. One, the GDP revision before the FOMC statement lowered expected interest rates and limited the Fed's ability to stimulate the economy. Two, the forward guidance was communicated at the same time households received information about a weaker economic outlook. A comparison of the forecasts following the GDP revision and the FOMC statement shows there was a larger decline in expected interest rates and a smaller decline in forecasts of real GDP after the FOMC statement. Figure 10 shows that when forward guidance is accompanied by a negative demand shock, it lowers expected interest rates and may cause real GDP forecasts to fall. Furthermore, real GDP is higher and expected nominal rates are lower than without forward guidance. Those results are consistent with the changes in the forecasts following the FOMC statement.

Statement	1	2	3	4	6	8	10
08/09/2011	−0.09	−0.18	−0.25	−0.29	−0.28	−0.20	−0.12
01/25/2012	−0.04	−0.10	−0.13	−0.14	−0.11	−0.05	0.01
09/13/2012	−0.01	−0.03	−0.05	−0.07	−0.06	−0.01	0.05

Table 6: Expected changes in forward rates j-years ahead on the date of the statement. Values are annualized net rates.

6.2 2012 POLICY STATEMENTS The January 25, 2012 and September 13, 2012 statements lengthened the forward guidance horizon for the federal funds rate. The January statement extended the horizon by six quarters (from mid-2013 to late-2014), but it was announced five quarters before the end of the August 2011 horizon. The September statement extended the horizon by two quarters (from late-2014 to mid-2015), six quarters before the January forward guidance ended.

The January and September 2012 FOMC statements only contained news that was intended to lower expected nominal rates beyond five quarters because the August 2011 statement already said the policy rate was likely to remain at its ZLB at least until mid-2013. Blue Chip forecasts, however, do not extend that far into the future. Thus, we use daily term structure data from Gürkaynak et al. (2007), which is regularly updated by the Board of Governors. Table 6 shows changes in instantaneous forward rates j-years ahead on the date of the FOMC statements. Following the January 2012 statement, the decline in the forward rates at 1 to 4 years ahead was about half the decline that occurred after the August 2011 statement. At longer horizons, the response is smaller and at 10 years ahead it is near zero. The September 2012 statement had an even smaller effect on future interest rates. Similarly, Raskin (2013) argues the August 2011 and January 2012 statements had different effects since the market was surprised by the first FOMC statement but not the second statement. Those results provide evidence that the central bank has a limited ability to affect future interest rates and stimulate the economy by extending the horizon, just like our theory predicts.

Date	2012Q2	2012Q3	2012Q4	2013Q1	2013Q2
BCFF (1/4-5)	2.14	2.30	2.66	2.59	2.72
BCEI (2/6-7)	2.21	2.40	2.59	2.51	2.68
Change	0.06	0.10	−0.07	−0.08	−0.04

(a) January 25, 2012 FOMC statement release.

Date	2012Q4	2013Q1	2013Q2	2013Q3	2013Q4
BCFF (9/5-6)	1.90	1.78	2.34	2.66	2.83
BCEI (9/24-25)	1.84	1.91	2.23	2.61	2.80
Change	−0.06	0.12	−0.12	−0.06	−0.03

(b) September 13, 2012 FOMC statement release.

Table 7: Blue Chip consensus forecasts of quarter-over-quarter real GDP growth. All values are annualized net rates.

Table 7 displays survey data analogous to what is shown in table 5. The data indicate the January and September FOMC statements had little effect on real GDP forecasts. The small marginal effect is consistent with our theory for two reasons. One, the August 2011 policy change reduced expected interest rates so much that the modest extension of the forward guidance horizon had a smaller margin to lower expected rates in order to stimulate real GDP. Two, the extension of the existing forward guidance horizon was less likely to be interpreted as news than the August 2011 announcement. Our results in figure 9 show modest extensions to the horizon can lead to a larger cumulative effect on real GDP, but most of that increase occurs at the end of the horizon. It is also possible concurrent information about a weak economy dampened real GDP forecasts, just like in figures 10 and 11a. Interestingly, a headline in the *New York Times* on the day of the January statement read, "Fed Signals That a Full Recovery Is Years Away." Such a reaction illustrates the challenge central banks face in achieving the desired effect of their forward guidance policies.[14]

7 CONCLUSION

This paper shows the forward guidance horizon, the state of the economy, the speed of the recovery, the degree of economic uncertainty, the expected stance of monetary policy, and the size of monetary policy shocks all nonlinearly impact the economic effects of forward guidance due to the ZLB constraint on current and future policy rates. We find the stimulative effect of forward guidance falls as the economy deteriorates or as households expect a slower recovery from a recession. Economic uncertainty has state-dependent effects. When the nominal rate is near but above zero, lower uncertainty increases the stimulative effect of forward guidance, whereas its stimulative effect is smaller when the notional rate is negative. Perhaps counterintuitively, a stronger monetary response to inflation also reduces the stimulus from forward guidance at the ZLB. A comparison of forward guidance to conventional monetary policy reveals that an unanticipated shock is more stimulative on impact than a news shock, but a news shock is more stimulative near the ZLB and always has a larger cumulative effect on real GDP. Lengthening the forward guidance horizon increases the cumulative effect on real GDP over short horizons but not over longer horizons. These results indicate that central banks face limits on the stimulus they can provide with forward guidance, but that stimulus is largest when the news is communicated early in an economic downturn.

Empirical estimates indicate that recent FOMC forward guidance reduced expected interest rates. It is unclear, however, how much of that decline was due to forward guidance and how much was due to changes in current and expected economic conditions. Overall, we find that recent forward guidance was often associated with declines in real GDP forecasts. Those outcomes were likely due to the fact that news was often accompanied by weak economic assessments and prior expectations of a weak economy gave policymakers a small margin to lower expected policy rates.

Our findings provide a solid foundation for future research on forward guidance. For example, one could examine the welfare gains from forward guidance in an economy where households learn about the monetary policy rule over time instead of knowing it with certainty. Another possibility is to assume the news process depends on future discount factor shocks. In that case, central bank communication would depend directly on the future state of the economy, which would provide a new way to model threshold-based forward guidance. It would also be interesting to examine various forms of communication about exiting the ZLB, especially given the recent rate increase.

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[14] Walsh (2009) cautions that aggressively reducing the policy rate in response to adverse shocks may cause a downward revision in people's economic outlook when their information set differs from the central bank. Campbell et al. (2012) suggest that real GDP declined in response to recent forward guidance because forecasters believed the Fed's communication was based on information about future economic conditions that was not available to the public. Bullard (2012) and Woodford (2012) argue date-based forward guidance may cause people to expect worse economic conditions over its horizon, whereas threshold-based forward guidance alleviates that problem by linking policy rate changes to economic conditions. Yellen (2013, 2014) refers to that type of communication as an "automatic stabilizer."

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A TWO-QUARTER HORIZON RESULTS

This section examines how the results in figure 1 change when the forward guidance horizon is extended by one quarter. Figure 13 plots the 2-quarter forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$, dashed line] decision rules for real GDP and the current and expected nominal interest rates as a function of the monetary policy shock, $\hat{\varepsilon}_t$. With 2-quarter forward guidance, households receive news about a policy shock two periods before the shock hits the economy. As a reference, we also show the decision rules without forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$, solid line]. Once again, we focus on a cross section of the decision rules where the initial notional interest rate equals zero.

When households receive news in period t about an expansionary monetary policy shock that will occur in period $t + 2$, the impact on real GDP is similar to the impact with 1-quarter forward guidance. Given households prefer a smooth consumption path, the expectation of monetary stimulus in period $t + 2$ encourages households to raise their consumption not only in period $t + 2$, but also in periods t and $t + 1$. The higher consumption in those periods stimulates current real GDP.

Central banks, in practice, offset the feedback effects on current and expected future nominal interest rates by promising to keep the nominal rate at zero over the entire forward guidance horizon. Thus, figure 13 also shows the decision rules when households receive 2-quarter distributed forward guidance [$(\alpha_0, \alpha_1, \alpha_2) = (0.16, 0.125, 0.715)$, dash-dotted line]. Substantial differences exist between the two types of 2-quarter forward guidance. With 2-quarter distributed forward guidance, the central bank announces in period t that an expansionary monetary policy shock will occur in periods t, $t+1$, and $t+2$. The shocks in periods t and $t+1$, which are not present with 2-quarter forward guidance, hold the current nominal interest rate at zero and lower the expected rate in period $t + 1$. Those two additional policy shocks more than compensate for the smaller weight on the period $t+2$ news shock, so 2-quarter distributed forward guidance produces a slightly larger stimulative effect than the more heavily weighted news shock that occurs in period $t + 2$. For example, a -0.5% (-1%) shock announced in period t raises real GDP by 0.05 (0.12) percentage points more with 2-quarter distributed forward guidance than with 2-quarter forward guidance.

Extending the horizon from 1 to 2 quarters less than doubles the stimulative effect on real GDP. For example, a -0.5% (-1%) policy shock increases real GDP by 0.18 (0.25) percentage points with 1-quarter distributed forward guidance and by 0.20 (0.33) percentage points with 2-quarter distributed forward guidance. Thus, the extra quarter of news only raises real GDP by an additional 0.02 (0.08) percentage points, which shows that longer horizons have diminishing marginal effects.

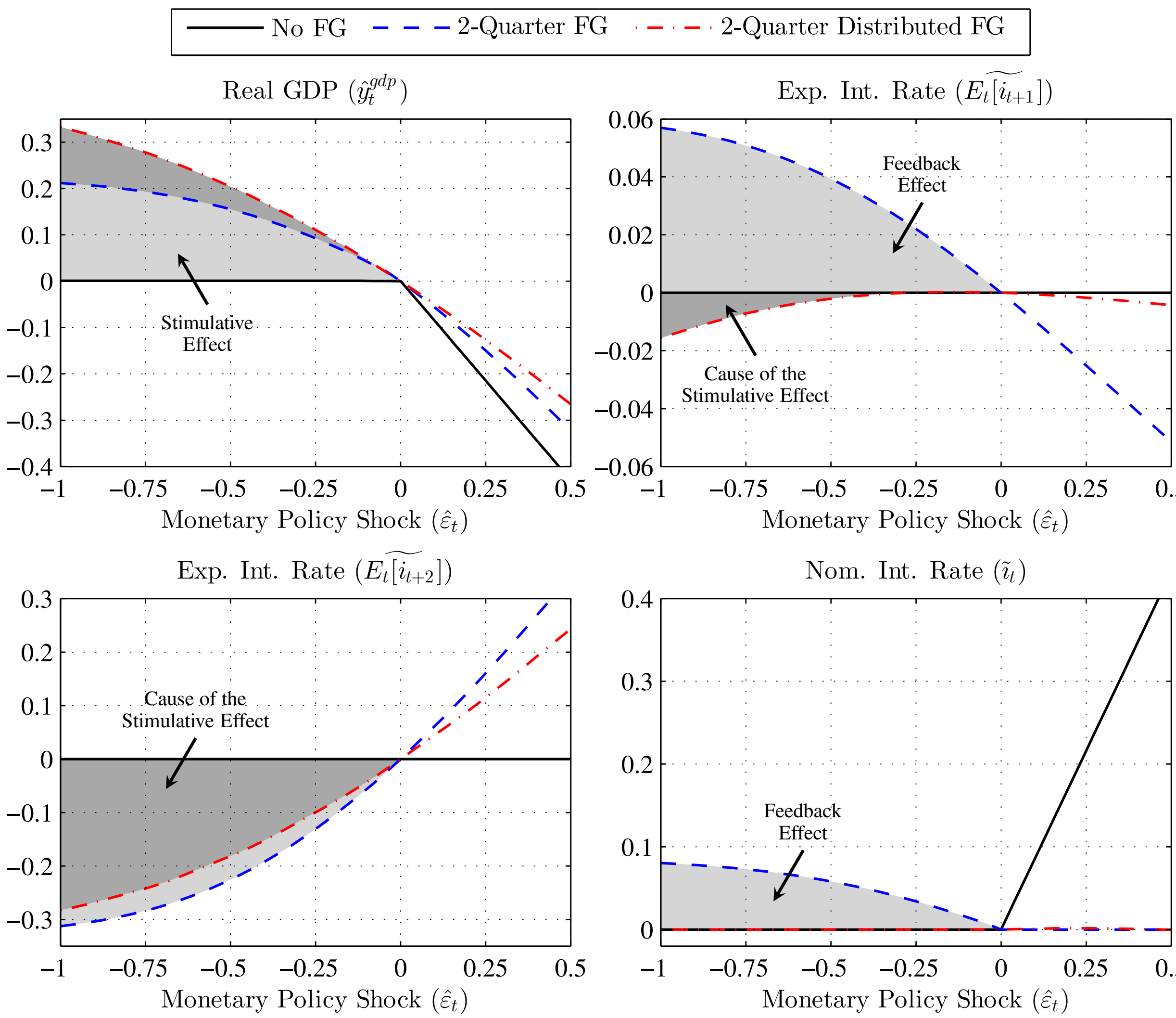
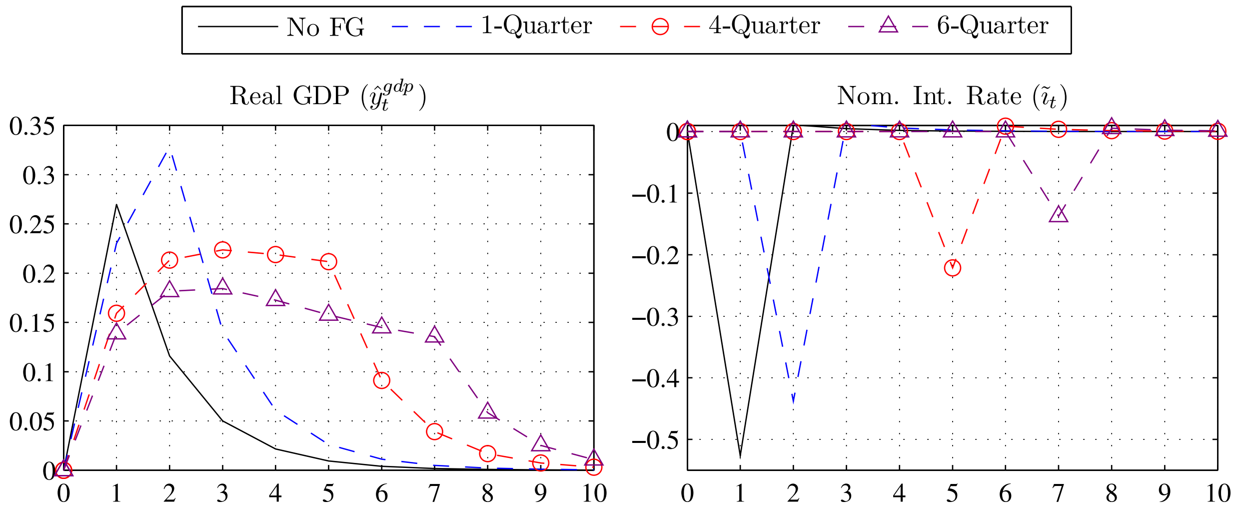


Figure 13: Decision rules as a function of the policy shock with no forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (1, 0, 0)$ (solid line); 2-quarter forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (0, 0, 1)$ (dashed line); and 2-quarter distributed forward guidance, $(\alpha_0, \alpha_1, \alpha_2) = (0.16, 0.125, 0.715)$ (dash-dotted line). In this cross section, the initial notional rate equals zero.

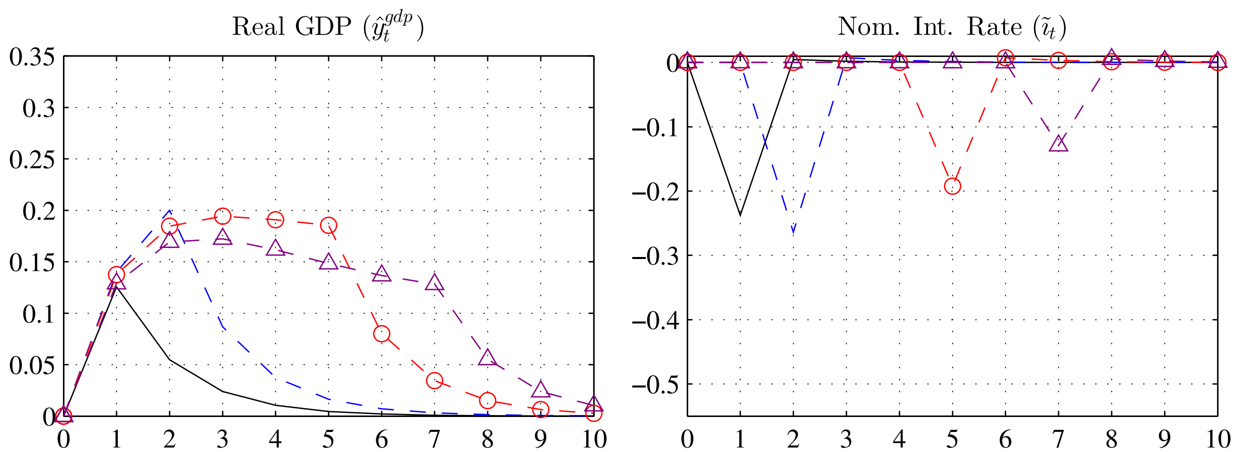
B MODEL WITH HABIT FORMATION

This section shows how the effects of forward guidance change when we extend the model in section 3 to allow for habit formation in the household's preferences—a feature many economists argue improves the model's empirical fit [e.g., Christiano et al. (2005) and Smets and Wouters (2007)]. A representative household chooses $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ to maximize $E_0 \sum_{t=0}^{\infty} \tilde{\beta}_t[\log(c_t - hc_{t-1}^a) - \chi n_t^{1+\eta}/(1+\eta)]$, where c^a is aggregate consumption, which is taken as given by the household, and h is the degree of external habit formation. The household's choices are constrained by $c_t + b_t = w_t n_t + i_{t-1} b_{t-1}/\pi_t + d_t$. The optimality conditions to the household's problem imply

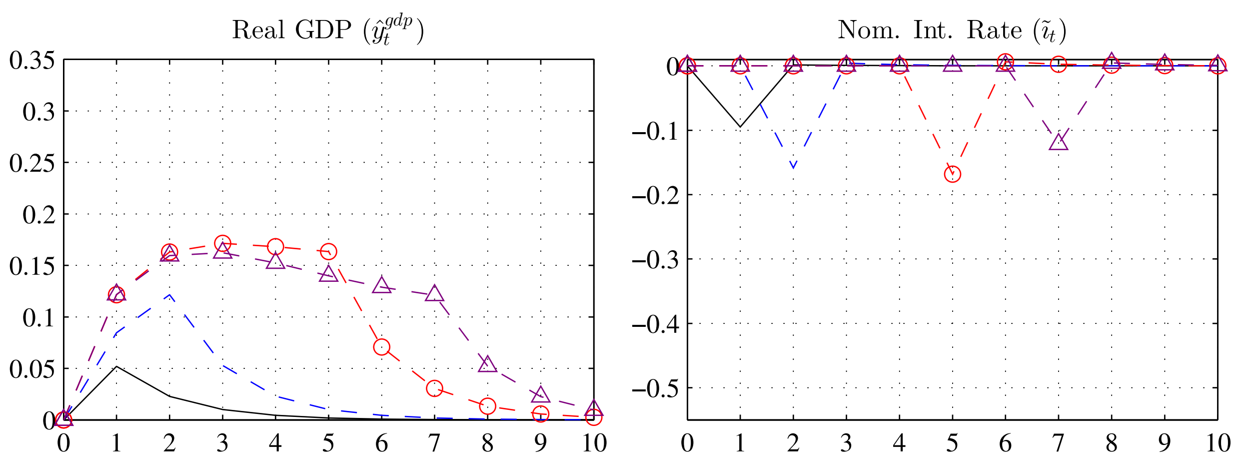
$$w_t = \chi n_t^{\eta}(c_t - hc_{t-1}^a), \quad (5)$$
$$1 = i_t E_t[q_{t,t+1}/\pi_{t+1}], \quad (6)$$



(a) Simulations initialized at steady state.



(b) Simulations initialized at a notional interest rate equal to zero.



(c) Simulations initialized at a notional interest rate equal to -0.5%.

Figure 14: Generalized impulse responses to a -0.6% monetary policy shock with no forward guidance, $(\alpha_0, \alpha_1) = (1, 0)$, (left panels) and distributed forward guidance (right panels). In each simulation, the weights on the distributed forward guidance shock (α_j, $j = 0, 1, \ldots, q$) are set to eliminate any feedback effects on the nominal interest rate.

where $q_{t,t+1} \equiv \beta_{t+1}(c_t - hc^a_{t-1})/(c_{t+1} - hc^a_t)$ is the pricing kernel between periods t and $t+1$ and $c_t = c^a_t$ in equilibrium. The production sector is unchanged, except firms now discount future dividends by $q_{t,k} \equiv \prod_{j=t+1}^{k>t} q_{j-1,j}$. When $h = 0$, the model is identical to the one in section 3. Gust et al. (2013) use a particle filter to estimate a constrained nonlinear model similar to this model. Thus, we set the habit formation parameter, h, to their mean posterior estimate of 0.46629.

Habit formation in consumption influences both the impact effect and duration of real GDP's response to forward guidance. Nevertheless, this paper's four key findings are unaffected by habit formation. Figure 14 shows generalized impulse responses to a -0.6% monetary policy shock distributed over 1-, 4-, and 6-quarter forward guidance horizons in the model with habit formation. The assumptions underlying the GIRFs are identical to figure 9 and are thus directly comparable.

	Forward Guidance Horizon							
	Without Habit Formation				With Habit Formation			
Initial State of the Economy	0	1	4	6	0	1	4	6
Steady State ($\tilde{\imath}^*_0 = 1$)	0.50	0.83	1.19	1.21	0.47	0.80	1.17	1.19
Recession ($\tilde{\imath}^*_0 = 0$)	0.23	0.51	1.00	1.08	0.22	0.48	0.97	1.06
Deep Recession ($\tilde{\imath}^*_0 = -0.5$)	0.11	0.33	0.87	0.99	0.09	0.29	0.84	0.97

Table 8: Present value of the cumulative percent change in real GDP in response to a -0.6% monetary policy shock.

There are four important differences in the responses compared to the model without habit formation. One, the impact effect of forward guidance is much smaller. Two, the peak response of real GDP is delayed such that real GDP increases gradually until about half way through the forward guidance horizon. Three, the stimulative effect of forward guidance lasts beyond the forward guidance horizon, although its post-horizon effect is small relative to real GDP's response in each quarter over the horizon. Four, despite being more persistent, table 8 shows the cumulative effect of forward guidance at each horizon and each initial notional interest rate is slightly smaller.

Essentially, the presence of habit formation breaks the link between consumption growth and the real interest rate, so current real GDP is less sensitive to changes in current and expected future real interest rates. Therefore, a distributed news shock, which simultaneously eliminates the feedback effect on the nominal interest rate and pushes up inflation, causes real GDP to peak on impact in our model without habit formation but is delayed in our model with habit formation.

C NUMERICAL ALGORITHM

A formal description of the numerical algorithm begins by writing the model compactly as

$$\mathbb{E}[f(\mathbf{v}_{t+1}, \mathbf{w}_{t+1}, \mathbf{v}_t, \mathbf{w}_t)|\Omega_t] = 0,$$

where f is a vector-valued function that contains the equilibrium system, $\mathbf{v} = \beta$ is a vector of exogenous variables, $\mathbf{w} = (c, n, y, w, \pi, i)$ is a vector of endogenous variables, and $\Omega_t = \{S, P, \mathbf{z}_t\}$ is the household's information set in period t, which contains the structural model, S, its parameters, P, and the state vector, $\mathbf{z}$. For example, with 1-quarter distributed forward guidance, $\mathbf{z}_t = (\varepsilon_{t-1}, \varepsilon_t, \beta_t)$. Each state variable is discretized into 61 points, so the state space contains $226{,}981$ nodes. The bounds of each state variable are ± 4 standard deviations of their processes.

The following steps outline our policy function iteration algorithm:

1. Obtain initial conjectures for the approximating functions, $\hat{c}_0$ and $\hat{\pi}_0$, on each node from the log-linear model without the ZLB imposed. We use `gensys.m` to obtain those conjectures.
2. For iteration $i \in \{1, \ldots, I\}$ and node $n \in \{1, \ldots, N\}$, implement the following steps:
   (a) On each node, solve for $\{y_t, i_t, w_t\}$ given $\hat{c}_{i-1}(\mathbf{z}_t^n)$ and $\hat{\pi}_{i-1}(\mathbf{z}_t^n)$ with the ZLB imposed.
   (b) Linearly interpolate $\{c_{t+1}, \pi_{t+1}\}$ given the state, $\{\varepsilon_t, \varepsilon_{t+1}^m, \beta_{t+1}^m\}_{m=1}^M$ (1-quarter forward guidance). Each of the M pairs of $\{\varepsilon_{t+1}^m, \beta_{t+1}^m\}$ are Gauss-Hermite quadrature nodes. In a constrained model, the accuracy of expectations is crucial, so we use 31 nodes on each shock ($M = 31^2$). We use Gauss-Hermite quadrature, since it is accurate for normally distributed shocks. We use piecewise linear interpolation to approximate future variables that show up in expectation, since that approach more accurately captures the kink in the decision rules than continuous functions such as Chebyshev polynomials.
   (c) On each node, solve for time $t+1$ variables, $\{y_{t+1}^m, c_{t+1}^m\}_{m=1}^M$, that enter the expectation operators. Then, numerically integrate to approximate the expectations by computing

$$\mathbb{E}\left[f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n)|\Omega_t\right] \approx \frac{1}{\pi}\sum_{m=1}^{M} f(\mathbf{x}_{t+1}^m, \mathbf{x}_t^n)\phi(\varepsilon_{t+1}^m, \beta_{t+1}^m),$$

   where $\mathbf{x} \equiv (\mathbf{z}, \mathbf{w})$, and ϕ are the respective Gauss-Hermite weights. The superscripts on $\mathbf{x}$ indicate which realizations of the state variables are used to compute expectations. Finally, use the nonlinear solver, `csolve.m`, to minimize the Euler equation errors.
3. Define $\text{maxdist}_i \equiv \max\{|\hat{c}_i - \hat{c}_{i-1}|, |\hat{\pi}_i - \hat{\pi}_{i-1}|\}$. Repeat step 2 until $\text{maxdist}_i < 10^{-9}$ on every node for 10 consecutive iterations. At that point, the algorithm converged to a solution.

Richter et al. (2014) demonstrate the accuracy of this algorithm in a model with a ZLB constraint.

D GENERALIZED IMPULSE RESPONSE FUNCTIONS

The general procedure for calculating GIRFs is described in Koop et al. (1996). The GIRFs are based on the average path from repeated simulations of our model and generated by following:

1. Initialize each simulation by solving for the constant discount factor shock that yields the desired notional interest rate. Define the corresponding state vector as $\mathbf{z}_0$.
2. Draw random monetary policy and discount factor shocks, $\{\varepsilon_t, \upsilon_t\}_{t=0}^N$, for each simulation, where N is the number of quarters in the simulation. Beginning at the initial state vector, $\mathbf{z}_0$, simulate R equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0)\}_{t=0}^N$, where $j \in \{1, 2, \ldots, R\}$ and $R = 100{,}000$.
3. Using the same R draws of shocks from step 2, replace the policy rate shock in period one with a -0.5% shock (i.e., set $\varepsilon_1 = -0.5$ for all $j \in \{1, 2, \ldots, R\}$). Then simulate the model with these alternate sequences of shocks to obtain R equilibrium paths, $\{\mathbf{x}_t^j(\mathbf{z}_0, \varepsilon_{z,1})\}_{t=0}^N$.
4. Average across the R simulations from step 2 and step 3 to obtain average paths given by

$$\bar{\mathbf{x}}_t(\mathbf{z}_0) = R^{-1}\textstyle\sum_{j=1}^R \mathbf{x}_t^j(\mathbf{z}_0), \qquad \bar{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1}) = R^{-1}\textstyle\sum_{j=1}^R \mathbf{x}_t^j(\mathbf{z}_0, \varepsilon_{z,1}).$$

5. The difference between the two paths is a GIRF. In our figures, a variable with a hat equals $100(\bar{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1})/\bar{\mathbf{x}}_t(\mathbf{z}_0) - 1)$, and a variable with a tilde is $100(\bar{\mathbf{x}}_t(\mathbf{z}_0, \varepsilon_{z,1}) - \bar{\mathbf{x}}_t(\mathbf{z}_0))$.

E Computing Longer Horizons

To make our numerical algorithm tractable across forward guidance horizons up to 10 quarters, we discretize each monetary policy shock with 3 points by following the procedure in Tauchen (1986). The state vector, $\mathbf{z}_t = (\beta_t, s_{0,t}, s_{1,t})$, is independent of the horizon. The monetary policy state, $s_{0,t} \in \{0, 1, 2\}$, determines the realization of the monetary policy shock, ε_t, according to

$$\varepsilon_t = \begin{cases} -0.006 & \text{for} \quad s_{0,t} = 0 \\ 0 & \text{for} \quad s_{0,t} = 1 \\ 0.006 & \text{for} \quad s_{0,t} = 2 \end{cases}.$$

A particular realization of the lagged monetary policy states in the news process is given by $\mathbf{e}(s_{1,t}, q) = [s_{1,t} \bmod 3, \lfloor s_{1,t}/3 \rfloor \bmod 3, \ldots, \lfloor s_{1,t}/3^{q-1} \rfloor \bmod 3]$, where $s_1 \in \{0, 1, \ldots, 3^q - 1\}$. The matrix of all realizations of lagged states is $E(q) \equiv [\mathbf{e}(s_1, q)]$. For example, when $q = 2$

$$E(2) = \begin{bmatrix} 0 & 1 & 2 & 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 2 \end{bmatrix}',$$

where the first (second) column of E corresponds to the state underlying the realization of ε_{t-1} (ε_{t-2}). The evolution of the state of lagged policy shocks is given by $s_{1,t+1} = s_{0,t} + 3(s_{1,t} \bmod 3^{q-1})$. If we further suppose $s_{0,t} = 2$ and $s_{1,t} = 3$, then $(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}) = (0.006, -0.006, 0)$. In order for $s_{1,t+1}$ to be consistent with the history of shocks, it must equal 2, which is given by $2 + 3(3 \bmod 3)$, so $(\varepsilon_t, \varepsilon_{t-1}) = (0.006, -0.006)$ (i.e., the third row of E).

The transition matrix for $s_{0,t}$ is ergodic and is characterized by a single vector of probabilities,

$$P = (\lambda_1, \lambda_2, \lambda_3) = (0.1587, 0.6827, 0.1587),$$

where $\lambda_k = \Pr(s_{0,t+1} = k)$. We discretize the initial discount factor, β_t, into 61 points, so the state space contains $N = 61 \times 3 \times 3^q$ nodes. We approximate the expectation operators by computing

$$\mathbb{E}\left[f(\mathbf{x}_{t+1}^k, \mathbf{x}_t^n) | \Omega_t\right] \approx \frac{1}{\pi} \sum_{k=1}^{3} \lambda_k \sum_{m=1}^{M} f(\mathbf{x}_{t+1}^{m,k}, \mathbf{x}_t^n) \phi(\beta_{t+1}^m),$$

where k is the realization of $s_{0,t+1}$. In all other aspects, the algorithm is the same as in appendix C.

For each horizon, we set the weights on the shocks ($\alpha_i, i = 0 \ldots, q$), so there are no feedback effects on the policy rate. Therefore, the weights are dependent on both the state of the economy and the forward guidance horizon. Table 9 reports the weights for each case shown in the paper.

F Computing Interest Rate Pegs

To model forward guidance as an interest rate peg, suppose the nominal interest rate is determined endogenously when $e_t = 0$ and is exogenously pegged at its ZLB when $e_t = 1$. Forward guidance policy is characterized by a vector of interest rate policies, $[e_t, e_{t+1}, \ldots, e_{t+q}]$, communicated to households in period t over horizon q. The state of forward guidance is s_t and a particular forward guidance policy is given by $\mathbf{f}(s_t, q) = [s_t \bmod 2, \lfloor s_t/2 \rfloor \bmod 2, \ldots, \lfloor s_t/2^q \rfloor \bmod 2]$, where $s_t \in$

q	α_0	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}
Steady State ($\tilde{\imath}_0^* = 1$)											
1	0.173	0.827	0	0	0	0	0	0	0	0	0
4	0.206	0.161	0.123	0.089	0.422	0	0	0	0	0	0
8	0.199	0.161	0.130	0.105	0.083	0.065	0.050	0.036	0.170	0	0
10	0.194	0.158	0.129	0.106	0.085	0.068	0.054	0.042	0.032	0.023	0.110
ZLB ($\tilde{\imath}_0^* = 0$)											
1	0.131	0.869	0	0	0	0	0	0	0	0	0
4	0.189	0.152	0.119	0.088	0.453	0	0	0	0	0	0
8	0.193	0.158	0.129	0.104	0.084	0.066	0.050	0.037	0.180	0	0
10	0.190	0.156	0.128	0.104	0.085	0.068	0.055	0.043	0.039	0.024	0.115
Deep ZLB ($\tilde{\imath}_0^* = -0.5$)											
1	0.113	0.887	0	0	0	0	0	0	0	0	0
4	0.184	0.149	0.116	0.086	0.465	0	0	0	0	0	0
8	0.191	0.157	0.128	0.104	0.083	0.066	0.051	0.037	0.183	0	0
10	0.189	0.155	0.127	0.104	0.085	0.068	0.055	0.043	0.033	0.024	0.117

Table 9: Weights on the monetary policy shock ($\alpha_i, i = 0 \ldots, q$) in each state of the economy.

$\{0, \ldots, 2^{q+1} - 1\}$. The matrix of all policies is defined by $F(q) \equiv [\mathbf{f}(s_t, q)]$. The forward guidance state, s_t, evolves according to a 2^{q+1}-state Markov chain with a transition matrix given by

$$P(q) \equiv [\underbrace{I_{2^q} \otimes [p, p]'}_{2^{q+1} \times 2^q}, \underbrace{I_{2^q} \otimes [1-p, 1-p]'}_{2^{q+1} \times 2^q}].$$

For example, with a 1-quarter forward guidance horizon,

$$F(1) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad P(1) = \begin{bmatrix} p & 0 & 1-p & 0 \\ p & 0 & 1-p & 0 \\ 0 & p & 0 & 1-p \\ 0 & p & 0 & 1-p \end{bmatrix},$$

where the first (second) column of F corresponds to the possible realizations of e_t (e_{t+1}). In state 0 ($[e_t, e_{t+1}] = [0, 0]$), the nominal interest rate is endogenous in periods t and $t+1$. Thus, economic conditions and not an exogenous interest rate peg determine whether the ZLB binds. The probability that forward guidance remains in state 0 is p, whereas $1-p$ is the probability that forward guidance will enter state 2 ($[e_t, e_{t+1}] = [0, 1]$). In state 2, the period t nominal interest rate is still set endogenously, but the central bank credibly announces the period $t+1$ nominal rate will be pegged to $\underline{\imath}$ regardless of economic conditions. That promise exogenously sets i_{t+1}, whereas news shocks allow for the possibility that $i_{t+1} > \underline{i}$. Forward guidance then transitions from state 2 to state 1 ($[e_t, e_{t+1}] = [1, 0]$) with probability p such that the nominal interest rate is pegged in period t but is endogenously set in period $t+1$. Alternatively, a $1-p$ probability exists that forward guidance moves from state 2 to state 3 ($[e_t, e_{t+1}] = [1, 1]$), which extends the interest rate peg by one quarter. In that case, households only know with certainty that the peg will last one quarter, although it may actually last for several quarters. With a longer forward guidance horizon ($q > 1$), households would expect the central bank to peg the nominal interest rate for more periods.