

Cyclical Net Entry and Exit*

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ABSTRACT

This paper examines how the interplay between cyclical net entry and exit of firms and search and matching frictions affect business cycle dynamics. We show cyclical net entry and exit reallocates inputs across firms and destroys jobs in recessions, which amplifies and skews business cycle dynamics. The model matches the volatility and skewness of real activity, the fast rise and slow decline in unemployment that occurs in recessions, and the counter-cyclical variation in macroeconomic uncertainty. Cyclical net entry and exit generates a 20% increase in volatility, 40% increase in skewness, and 55% increase in the welfare cost of business cycles.

Keywords: Firm Entry; Firm Exit; Unemployment; Nonlinear; Skewness; Welfare; Uncertainty

JEL Classifications: E24; E32; E37; J63; L11

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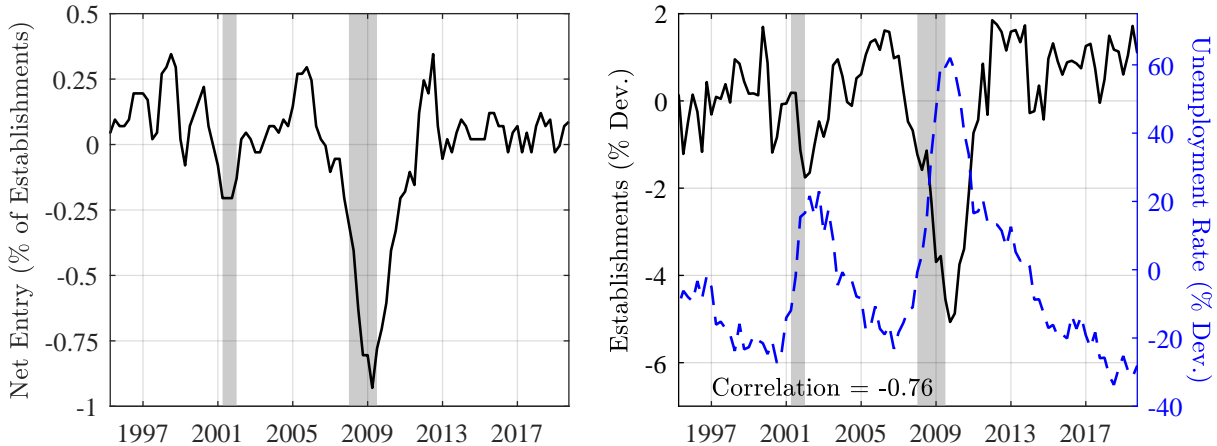


Figure 1: The left panel plots a 4-quarter moving average of establishment net entry as a percent of all establishments (demeaned) using data from the Business Employment Dynamics database. The right panel plots the unemployment rate and establishments as percent deviations from trend using a Hamilton (2018) filter.

1 INTRODUCTION

Standard aggregate analyses of business cycles abstract from cyclical net entry and exit of firms. While this omission may be justified during periods of economic tranquility, [Figure 1](#) shows there was substantial net exit of firms in the U.S. during the Great Recession. The net exit of firms occurred alongside a large spike in unemployment and drop in real activity. Motivated by these infrequent but significant events, this paper highlights the amplification, asymmetry, and welfare costs that result from the interplay between net firm entry and endogenous job creation and destruction.

To conduct our analysis, we generalize a textbook search and matching model with risk-averse households and capital to include multi-worker firms that endogenously enter and exit.¹ Each firm faces decreasing returns to scale in production, which generates non-zero profits and creates motives for entry and exit in equilibrium.² This straightforward model extension permits a global nonlinear solution, which is essential to accurately quantify the effects of net entry and exit on the business cycle. Our results are also directly comparable to other aggregate business cycle models.

We analytically highlight two effects of the interaction between cyclical net entry and exit and frictional labor markets. First, net entry and exit amplify the effects of technology shocks on the labor market and output, consistent with [Jaimovich and Floetotto \(2008\)](#), [Bilbiie et al. \(2012\)](#) and [Clementi and Palazzo \(2016\)](#). Second, job destruction due to net firm exit amplifies recessions and creates asymmetric business cycle dynamics. Consider a negative technology shock, which lowers output and firm profits. This causes some firms to exit, which lowers productivity as inputs are reallocated to surviving firms who face decreasing returns to scale. With fewer firms, each with

¹A “firm” refers to an establishment where multiple workers are employed (e.g., an office, factory, or headquarters).

²An alternative, but qualitatively identical approach, is to assume each firm produces a unique good in a monopolistically competitive environment. See [Jaimovich and Floetotto \(2008\)](#) and [Bilbiie et al. \(2012\)](#) for recent examples.

lower productivity, output falls by more than in a model without entry and exit. Specific to our framework, vacancy creation also declines more due to the lower marginal product of labor, which leads to a larger drop in unemployment than without net entry and exit. This drop is further amplified by the net exit of firms, which directly destroys jobs and lowers output. Since net exit only occurs in downturns, this link between firm exit and job destruction causes asymmetric dynamics.

To quantify the strength of these mechanisms, we calibrate the model parameters governing firm entry and exit to target the shares of job creation and job destruction due to net entry and exit of establishments in the data. Other parameters are set to match a range of benchmark moments. As validation, we confirm that our model is consistent with the volatility and skewness of net entry and exit, and its correlations with real activity. To understand the effects of cyclical net entry and exit, we compare our model outcomes to the data and a model with only search and matching frictions.

We measure amplification by comparing the standard deviations of macroeconomic aggregates in the models with and without cyclical net entry and exit. While the baseline model generates realistic volatilities of output, consumption, investment, and unemployment, removing entry and exit reduces volatility by about 20% across all measures of real activity. A similar story applies to the asymmetry mechanism, which we measure by comparing skewness across models and the data. Although untargeted, our baseline model generates realistic skewness in all variables. Furthermore, the model generates positive skewness in unemployment growth, which is a common proxy for the fast rises and slow declines in the unemployment rate (Ferraro, 2018; Neftci, 1984). In contrast, the model without entry and exit generates 40% less skewness in real activity and no asymmetry in unemployment growth, as noted by Ferraro (2017). These results highlight the important role of net firm exit and job destruction in generating the sudden and deep recessions that occur in the data.

Net exit and the ensuing endogenous job destruction also generates state-dependence in the responses to technology shocks, consistent with empirical results in Pizzinelli et al. (2020). In our model, a decrease in technology reduces profits causing some firms to exit. This leads to an increase in job destruction, which creates a fast-slow dynamic in unemployment as separated workers go through the search and matching process. The amount of exit depends on the state of the economy.

We emphasize two consequences of our model's non-Gaussian dynamics. First, households would be willing to give up 0.42% of lifetime consumption to forgo business cycle fluctuations. This welfare cost is 8 times larger than the calculation in Lucas (2003) that assumes a Gaussian consumption process and 55% larger than the cost in the model without entry and exit. This is driven by the additional negative skewness that entry and exit imparts to aggregate consumption.

Second, our model is consistent with recent empirical work that finds macroeconomic uncertainty is often an endogenous response to exogenous first moment shocks rather than an exogenous propagation (Ludvigson et al., 2020). Intuitively, the asymmetry mechanism in the model generates a negatively skewed distribution of future output, which is particularly pronounced in recessions.

The state-dependent transmission of aggregate shocks allows the model to match the counter-cyclical variation in macroeconomic uncertainty without the aid of exogenous volatility shocks.

Related Literature Our analysis sits within the class of aggregate models that examine cyclical net entry and exit of firms in real business cycle settings. Our contributions are to link cyclical net entry and exit to unemployment via search and matching frictions and explore the higher-order properties of our model, including the consequences for welfare and macroeconomic uncertainty.

A few earlier papers examine cyclical net entry and exit in log-linear models without labor market search. As such, they abstract from higher-order moments and omit the link between endogenous firm exit and job destruction that we show creates asymmetric macroeconomic dynamics. A first class of models use a zero profit condition to model endogenous entry and exit without distinguishing between incumbents and entrants (Chatterjee and Cooper, 1993; Devereux et al., 1996a,b). Building on this work, Jaimovich and Floetotto (2008) find cyclical variation in the number of competing firms leads to counter-cyclical markups and pro-cyclical variation in productivity. A second class of models make the distinction between incumbents and entrants, but abstract from endogenous exit. For example, Bilbiie et al. (2012) show that endogenous product entry amplifies real business cycles.³ Our model features endogenous entry and exit while also distinguishing between incumbents and entrants, allowing us to match the shares of job creation and job destruction due to entry and exit. Furthermore, none of these earlier papers compute the welfare cost of cyclical net entry and exit, nor do they draw attention to the consequences for macroeconomic uncertainty.⁴

The literature on search and matching frictions in a business cycle setting is extensive. Our analysis maintains the quantitative tradition of the early literature (Andolfatto, 1996; Den Haan et al., 2000; Merz, 1995), while incorporating the insights of the recent literature that abstracts from capital and risk aversion (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017; Mortensen and Nagypal, 2007; Shimer, 2005). Our results also capture the nonlinear congestion externality described in Petrosky-Nadeau and Zhang (2017) and Petrosky-Nadeau et al. (2018).⁵

To quantify the nonlinear interactions in our model, we compute the volatility, skewness, and uncertainty of key macro variables. These statistics complement the literature that documents empirical departures from normality. Neftci (1984) was the first to provide formal evidence that the U.S. unemployment rate rises faster in recessions than it declines in expansions. Sichel (1993) finds there is not only asymmetry in unemployment growth (steepness asymmetry), but also in the level of unemployment (deepness asymmetry). Acemoglu et al. (2017) find the distribution of U.S. out-

³Hamano and Zanetti (2017, 2020), Colciago and Silvestrini (2020), and Hartwig and Lieberknecht (2020) use similar models with endogenous firm entry and exit, but they do not include endogenous job creation and destruction.

⁴Recently, Bilbiie et al. (2019) compute the welfare costs of the firm entry distortions in Bilbiie et al. (2012). They also find large welfare costs, but in our model the costs are driven by the skewness from endogenous job destruction.

⁵Search and matching models in which firms employ one worker technically feature entry and exit. However, this setting is difficult to map to the data, and it is silent on how the number of firms interacts with labor market frictions.

put growth rates is negatively skewed and features more mass in the tails than a normal distribution implies. Finally, Bekaert and Popov (2019) document that similar patterns hold outside of the U.S.

Recent papers emphasize various mechanisms that generate the non-normal features of macro data. Closest to us, Ferraro (2018) and Pizzinelli et al. (2020) develop models of heterogeneous jobs in which low productivity workers are endogenously separated when aggregate productivity falls below a threshold. In our model, the firm exit decision also follows a productivity-driven threshold rule, but firm exit causes the separation of every worker at an exiting firm. We discipline our approach using data on the share of job destruction due to firm exit and show it is a powerful source of asymmetry. Relatedly, McKay and Reis (2008) explain that contractions in employment are briefer and more violent than expansions using a model with asymmetric employment adjustment costs and endogenous technology adoption. Dupraz et al. (2019) show downward wage rigidity generates negative skewness in employment and asymmetry in the speeds of recessions and recoveries.⁶ Ilut et al. (2018) find the volatility and skewness in employment dynamics follows from asymmetry in how firms respond to information. Finally, Ferraro and Fiori (2020) show that search and matching frictions generate nonlinearities that explain the state-dependent effects of tax policy.

Our focus on cyclical net entry and exit in an aggregate setting complements a recent literature that uses heterogeneous firm models (Hopenhayn and Rogerson, 1993; Hopenhayn, 1992) to study how the gross entry and exit of firms interacts with labor market frictions and the business cycle.⁷ For example, Kaas and Kircher (2015) present a tractable heterogeneous firm model consistent with the firm size distribution and worker flows.⁸ They find firm heterogeneity has little impact on the business cycle relative to an aggregate model. In contrast, we find a meaningful effect of cyclical net entry and exit in our aggregate model when we focus on nonlinearities. These findings relate to Sedlacek (2020), who shows the lack of new firm startups during the Great Recession slowed the recovery. Finally, our results complement Schaal (2017), who shows that idiosyncratic uncertainty shocks account for 40% of the increase in unemployment during the Great Recession. We show our model can match the counter-cyclical variation in uncertainty without exogenous volatility shocks.

The paper proceeds as follows. [Section 2](#) lays out our model. [Section 3](#) highlights the amplification and asymmetry mechanisms using a simplified version of our model. [Section 4](#) describes our nonlinear solution method and calibration strategy. [Section 5](#) documents our model’s quantitative properties. [Section 6](#) examines the implications for welfare and uncertainty. [Section 7](#) concludes.

⁶Andolfatto (1997) was one of the first to document that a simple model with a search and matching mechanism can generate the observed asymmetries in the unemployment rate because it features an asymmetric job destruction rate.

⁷In this framework, Campbell (1998) develops a model where entry and exit is due to technological obsolescence.

⁸The model in Elsby and Michaels (2013) is similar to Kaas and Kircher (2015) except it does not feature endogenous firm entry. Clementi and Palazzo (2016) include endogenous entry and exit of heterogeneous firms in a model without labor market frictions. They emphasize that firm dynamics amplify technology shocks via input reallocation.

2 ENVIRONMENT

We use a textbook search and matching model augmented to include endogenous entry and exit of multi-worker firms. Households consume and accumulate capital, while firms produce to maximize profits and choose when to enter and exit the market. Frictional labor markets are intermediated by employment agencies who post vacancies and pay wages determined by Nash Bargaining.

2.1 SEARCH AND MATCHING Entering period t , there are N_{t-1} employed workers and U_{t-1} unemployed workers. Then a constant fraction \bar{s} of the employed workers exogenously lose their jobs. The remaining fraction $1 - \bar{s}$ endogenously lose their jobs with probability $1 - \xi_t$, which is the probability that a worker's firm exits the market. Thus, the endogenous job separation rate is given by

$$s_t = \bar{s} + (1 - \bar{s})(1 - \xi_t). \quad (1)$$

Newly separated workers search for a job in the same period as their job loss, but these workers have less time to search for a job than those who became unemployed in a previous period. Let $\chi \in [0, 1]$ denote the fraction of a period that newly unemployed workers search for work in the same period as their job loss. Then the number of unemployed searching for work in period t is given by

$$U_t^s = U_{t-1} + \chi s_t N_{t-1}. \quad (2)$$

Shimer (2005) sets χ equal to 0.5 when constructing a measure of the monthly job finding rate in the data. We obtain a similar value when calibrating χ to match the average unemployment rate.

Following Den Haan et al. (2000), the number of new matches in period t is given by

$$M_t = U_t^s V_t / ((U_t^s)^\iota + V_t^\iota)^{1/\iota},$$

where $\iota > 0$ determines the curvature of the matching function and V_t is vacancy postings. Define $\theta_t \equiv V_t / U_t^s$ as labor market tightness. The job-filling rate, q_t , and job-finding rate, f_t , are given by

$$q_t = M_t / V_t = 1 / (1 + \theta_t^\iota)^{1/\iota}, \quad (3)$$

$$f_t = M_t / U_t^s = \theta_t q_t. \quad (4)$$

Following Blanchard and Galí (2010), we assume newly matched workers begin employment in the same period they are matched with a firm, so aggregate employment evolves according to

$$N_t = (1 - s_t)N_{t-1} + q_t V_t. \quad (5)$$

The unemployment rate U_t includes anyone who is not employed in period t , so it is given by

$$U_t \equiv U_t^s - M_t = 1 - N_t. \quad (6)$$

2.2 HOUSEHOLDS A representative household is populated by a unit mass of workers who are either employed or unemployed, so the fraction of employed workers coincides with the aggregate employment rate. Following Merz (1995), Andolfatto (1996), and Den Haan et al. (2000), workers pool their incomes together to achieve perfect consumption insurance. Households rent capital, K_{t-1} , to firms. Given the specification in Jermann (1998), capital is subject to adjustment costs,

$$K_t = (1 - \delta)K_{t-1} + \left(a_1 + \frac{a_2}{1 - 1/\nu} \left(\frac{I_t}{K_{t-1}} \right)^{1-1/\nu} \right) K_{t-1} \quad (7)$$

where $0 < \delta \leq 1$ is the capital depreciation rate, $\nu > 0$ determines the size of the capital adjustment cost, and $a_1 = \delta/(1 - \nu)$ and $a_2 = \delta^{1/\nu}$ are chosen so there are no adjustment costs in steady state.

The household chooses consumption, C_t , and investment, I_t , to solve

$$J_t^H = \max_{C_t, I_t} \ln C_t + \beta E_t J_{t+1}^H$$

subject to (7) and

$$\begin{aligned} C_t + I_t &= w_t^n N_t + r_{k,t} K_{t-1} + D_t + bU_t - \tau_t, \\ N_{t+1} &= (1 - s_{t+1}(1 - \chi f_{t+1}))N_t + f_{t+1}U_t, \\ U_{t+1} &= s_{t+1}(1 - \chi f_{t+1})N_t + (1 - f_{t+1})U_t, \end{aligned}$$

where β is the discount factor, w_t^n is the wage rate, $r_{k,t}$ is the rental rate, D_t is dividends net of start up costs for entering firms and rebates of fixed production costs, b is the flow value of unemployment, τ_t is a lump sum tax, and E_t is the expectation operator conditional on period- t information.

The optimality conditions imply

$$\frac{1}{a_2} \left(\frac{I_t}{K_{t-1}} \right)^{1/\nu} = E_t \left[x_{t+1} \left(r_{k,t+1} + \frac{1}{a_2} \left(\frac{I_{t+1}}{K_t} \right)^{1/\nu} (1 - \delta + a_1) + \frac{1}{\nu - 1} \frac{I_{t+1}}{K_t} \right) \right], \quad (8)$$

where $x_{t+1} = \beta(C_t/C_{t+1})$ is the household's stochastic discount factor. This condition says the marginal cost of investing in period t equals the discounted marginal benefit in period $t + 1$, which includes the return on capital, the undepreciated capital stock, and the foregone adjustment costs.

2.3 FIRMS There are incumbent firms and potential entrants who are not currently producing.

Incumbent Firms Let Z_{t-1} denote the mass of incumbent firms entering period t . Each incumbent firm chooses whether to remain active or to exit.⁹ Conditional on remaining active, an incumbent chooses its capital and labor inputs $\{k_t, n_t\}$ to maximize profits using the technology $y_t = a_t(k_t^\alpha n_t^{1-\alpha})^\vartheta$ where $\alpha, \vartheta \in (0, 1)$. This specification follows the firm dynamics literature and uses decreasing returns to scale as a source of profits, which are necessary to generate motives for entry and exit (Bilal et al., 2019; Carvalho and Grassi, 2019; Clementi and Palazzo, 2016; Sedlacek, 2020). Profit maximization implies all active firms make the same capital and labor choices.

Technology is denoted by a_t , which is common across firms and evolves according to

$$\ln a_{t+1} = (1 - \rho_a) \ln \bar{a} + \rho_a \ln a_t + \sigma_a \varepsilon_{a,t+1}, \quad 0 \leq \rho_a < 1, \quad \varepsilon_a \sim \mathbb{N}(0, 1). \quad (9)$$

Define $J_{X,t}^F = \max\{J_{A,t}^F, 0\}$ as the value of an incumbent choosing to exit or remain active at the start of period t . Conditional on choosing to actively produce in period t , a firm's value satisfies

$$J_{A,t}^F = \max_{k_t, n_t} a_t(k_t^\alpha n_t^{1-\alpha})^\vartheta - w_t n_t - r_{k,t} k_t - \psi_y + E_t[x_{t+1} J_{X,t+1}^F],$$

where ψ_y is a fixed operating cost a firm pays to produce. Operating costs are rebated to the household. Exiting firms have no scrap value, which is a normalization given their fixed operating cost.

Potential Entrants There is a mass of firms who can become active after paying a one-time fixed cost $\psi_n \geq 0$. Therefore, the value of an inactive potential entrant firm is $J_{E,t}^F = \max\{J_{A,t}^F - \psi_n, 0\}$.

Net Entry and Exit Let $\Delta Z_t = Z_t - Z_{t-1}$ denote the net entry of firms in period t . The free entry condition, $J_{A,t}^F \leq \psi_n$, holds with equality when $\Delta Z_t > 0$, so entry occurs up to the point at which the cost of entry equals its benefit. Similarly, the free exit condition, $J_{A,t}^F \geq 0$, holds with equality when $\Delta Z_t < 0$. Hence $J_{A,t}^F \in [0, \psi_n]$ and $J_{E,t}^F = 0$. These features capture in a stylized manner the dynamics of net entry and exit in the data, which is often stable and close to zero but occasionally exhibits large drops when there is net firm exit during recessions (see [Figure 1](#)). Our approach also generalizes previous models. Jaimovich and Floetotto (2008) assume that $\psi_n = 0$, while Bilbiie et al. (2012) impose an exogenous exit rate and linearize their economy around a steady state in which $J_A^F = \psi_n$. We allow for endogenous exit and calibrate ψ_n to generate realistic firm dynamics.

Aggregation Aggregating over the Z_t firms' profit-maximizing capital and labor choices yields,

$$Y_t = a_t Z_t^{1-\vartheta} (K_{t-1}^\alpha N_t^{1-\alpha})^\vartheta, \quad (10)$$

$$w_t = (1 - \alpha)\vartheta Y_t / N_t, \quad (11)$$

$$r_{k,t} = \alpha\vartheta Y_t / K_{t-1}, \quad (12)$$

⁹An incumbent is a firm that exists at the start of period t . Any firm that produces in period t is considered active.

where $K_{t-1} = Z_t k_t$ is the aggregate capital stock and $N_t = Z_t n_t$ is aggregate employment. Similar to Clementi and Palazzo (2016), (10) shows the mass of incumbent firms is an input to production. Optimality conditions (11) and (12) yield an expression for per-firm flow profits $(1 - \vartheta)Y_t/Z_t - \psi_y$.

Given profit maximization, the aggregate value of active firms in period t satisfies

$$Z_t J_{A,t}^F = (1 - \vartheta)Y_t - Z_t \psi_y + Z_t E_t[x_{t+1} J_{X,t+1}^F] \quad (13)$$

where $J_{X,t+1}^F = \max\{J_{A,t+1}^F, 0\}$ is the continuation value of active firms. Optimal net entry and exit behavior implies Z_t and $J_{A,t}^F$ satisfy (13) and the following complementary slackness conditions:

$$\begin{cases} \min\{\Delta Z_t, 0\} J_{A,t}^F = 0, & J_{A,t}^F \geq 0, \\ \max\{\Delta Z_t, 0\} (J_{A,t}^F - \psi_n) = 0, & J_{A,t}^F \leq \psi_n. \end{cases} \quad (14)$$

When $J_{A,t}^F = 0$ and there is net exit ($\Delta Z_t < 0$), the amount of net exit is such that (13) holds with $J_{A,t}^F = 0$. When $J_{A,t}^F \in (0, \psi_n)$, there is neither net entry nor exit ($\Delta Z_t = 0$). Finally, when $J_{A,t}^F = \psi_n$ and there is net entry ($\Delta Z_t > 0$), the amount of net entry is such that (13) holds with $J_{A,t}^F = \psi_n$.

The fraction of incumbent firms who remain active is defined by

$$\xi_t = \mathbb{I}\{J_{A,t}^F > 0\} + (Z_t/Z_{t-1})\mathbb{I}\{J_{A,t}^F = 0\}, \quad (15)$$

so the fraction of exiting firms is $1 - \xi_t$. This determines the mass of endogenous separations at the worker level shown in (1). When incumbent firms exit, ξ_t declines, which increases job separations.

2.4 EMPLOYMENT AGENCIES Employment agencies supply labor to active firms by posting vacancies in the frictional labor market.¹⁰ They sell each unit of labor to active firms at the competitive rate w_t and then pay workers a wage w_t^n determined by Nash Bargaining (described below).

The representative employment agency solves

$$J_t^E = \max_{N_t, V_t} (w_t - w_t^n)N_t - \kappa V_t + E_t[x_{t+1} J_{t+1}^E] \quad (16)$$

subject to (5) and $V_t \geq 0$, where $\kappa > 0$ is the per-period vacancy posting cost. Optimality implies

$$\lambda_{N,t} = (\kappa - \lambda_{V,t})/q_t, \quad (17)$$

$$\lambda_{N,t} = w_t - w_t^n + E_t[x_{t+1}(1 - s_{t+1})\lambda_{N,t+1}], \quad (18)$$

$$\lambda_{V,t} V_t = 0, \quad \lambda_{V,t} \geq 0, \quad (19)$$

where $\lambda_{N,t}$ and $\lambda_{V,t}$ are the Lagrange multipliers on (5) and the inequality constraint. Thus, $\lambda_{N,t}$ is

¹⁰If active firms directly posted vacancies, the vacancy posting decision would depend on each firm's entire employment history. In addition, the wage bargaining process would be complicated by the presence of multi-worker firms.

the marginal surplus value of a new match to the agency at time t . Combining (17) and (18) yields

$$\frac{\kappa - \lambda_{V,t}}{q_t} = w_t - w_t^n + E_t[x_{t+1}(1 - s_{t+1})\frac{\kappa - \lambda_{V,t+1}}{q_{t+1}}], \quad (20)$$

which determines vacancy creation by equating the marginal cost of posting an additional vacancy with the marginal benefit of an additional worker. The benefit includes the time t profit from the new match plus the present value of the foregone vacancy posting cost net of time $t+1$ separations.

2.5 WAGES As noted by Hall (2005), the match surplus created by search and matching frictions leads to wage indeterminacy in the absence of additional model structure. Furthermore, Ljungqvist and Sargent (2017) emphasize that the choice and calibration of the wage determination mechanism has strong implications for the volatility of unemployment.¹¹ Thus, we follow the bulk of the literature and assume wages are determined via Nash bargaining between an employed worker and the employment agency. We calibrate the parameters of this wage protocol to ensure that the model generates realistic labor market volatility. To operationalize Nash bargaining, define the total surplus of a new match as $\Lambda_t = \lambda_{N,t} + J_{N,t}^H - J_{U,t}^H$, where $J_{N,t}^H$ and $J_{U,t}^H$ satisfy the envelope conditions

$$\begin{aligned} J_{N,t}^H &= w_t^n + E_t[x_{t+1}((1 - s_{t+1}(1 - \chi f_{t+1}))J_{N,t+1}^H + s_{t+1}(1 - \chi f_{t+1})J_{U,t+1}^H)], \\ J_{U,t}^H &= b + E_t[x_{t+1}(f_{t+1}J_{N,t+1}^H + (1 - f_{t+1})J_{U,t+1}^H)]. \end{aligned}$$

The equilibrium wage rate maximizes $(J_{N,t}^H - J_{U,t}^H)\eta\lambda_{N,t}^{1-\eta}$, where $\eta \in [0, 1]$ is the household's bargaining weight. Optimality implies $J_{N,t}^H - J_{U,t}^H = \eta\Lambda_t$ or, equivalently, $\lambda_{N,t} = (1 - \eta)\Lambda_t$. To derive the equilibrium wage, combine the two optimality conditions with $J_{N,t}^H$, $J_{U,t}^H$, and (18) to obtain

$$w_t^n = \eta(w_t + \kappa E_t[x_{t+1}(1 - \chi s_{t+1})\theta_{t+1}]) + (1 - \eta)b. \quad (21)$$

The household's wage rate in period t is a weighted average of the firm's value of a new match and the worker's outside option b . The firm's value of a new worker includes the additional output produced plus the discounted expected value of the worker net of separations that occur in period $t+1$.

2.6 EQUILIBRIUM Given (13), (16), and $\tau_t = bU_t$, the aggregate resource constraint is given by

$$C_t + I_t + \kappa V_t = Y_t. \quad (22)$$

The equilibrium includes infinite sequences of quantities $\{C_t, N_t, U_t, U_t^s, V_t, I_t, K_t, Y_t, J_{A,t}^F, Z_t, \xi_t, q_t, f_t, s_t\}_{t=0}^\infty$, prices $\{\lambda_{V,t}, w_t^n, w_t, r_{k,t}\}_{t=0}^\infty$, and exogenous variables $\{a_t\}_{t=1}^\infty$ that satisfy (1)-(15) and (19)-(22), given the initial conditions $\{N_{-1}, Z_{-1}, K_{-1}, a_{-1}\}$ and sequences of shocks $\{\varepsilon_{a,t}\}_{t=0}^\infty$.

¹¹Mortensen and Nagypal (2007), Hagedorn and Manovskii (2008), Costain and Reiter (2008), and Zanetti (2011) also extensively discuss the relevance of the joint surplus for the dynamic properties of search and matching models.

3 UNDERLYING MECHANISMS

This section develops intuition for the two channels in our model: (1) Entry and exit amplify the transmission of aggregate technology shocks to output and the labor market; (2) The destruction of jobs caused by endogenous firm exit creates asymmetry and negative skew in output dynamics.

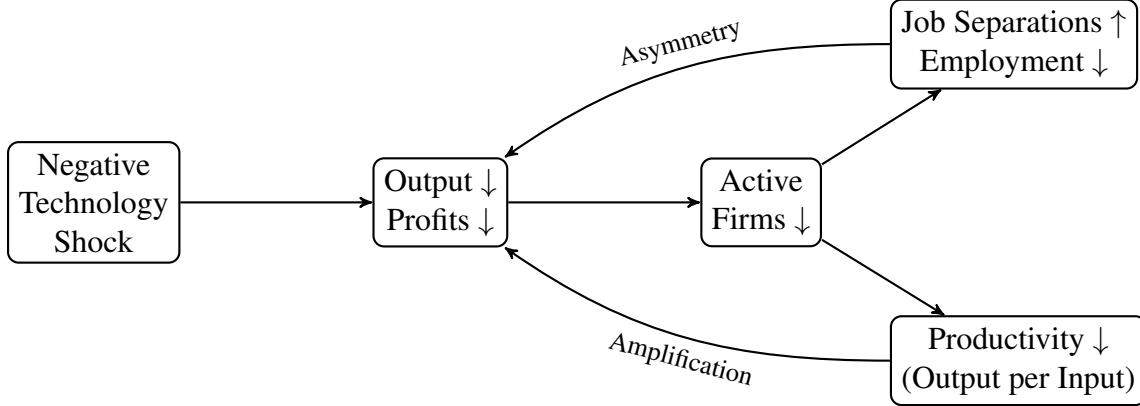


Figure 2: Endogenous sources of amplification and asymmetry due to cyclical net entry and exit.

Figure 2 illustrates how these mechanisms create endogenous feedback loops in the economy. To analyze the amplification and asymmetry channels, we consider a special case of our economy in which capital is fixed and the entry cost is zero ($\psi_n = 0$).¹² In this case, entry and exit ensure the value of active firms is zero ($J_{A,t}^F \equiv 0$). Applying these conditions to (13) yields an expression for the number of active firms, $Z_t = (1 - \vartheta)Y_t/\psi_y$, so the response of Z_t to a change in Y_t is given by

$$d \ln Z_t = d \ln Y_t. \quad (23)$$

Intuitively, the number of active firms is increasing in profits, which scale with aggregate output.

3.1 AMPLIFICATION To see the effects of changes in Z_t on output, differentiate (10) to obtain

$$d \ln Y_t = d \ln a_t + (1 - \vartheta)d \ln Z_t + \vartheta(1 - \alpha)d \ln N_t. \quad (24)$$

All else equal, (24) shows aggregate output is increasing in the number of active firms, and it responds more when there are stronger decreasing returns to scale (i.e., a lower ϑ). To understand the microeconomic foundation of this relationship, consider the case of net firm entry. When new firms enter, some of the aggregate capital and labor supply is reallocated from incumbents to entrants. Since each firm faces decreasing returns to scale, the decline in inputs per firm causes an increase in each firm's productivity, $y_{jt}/(k_{jt}^\alpha n_{jt}^{1-\alpha})$, and a boost in each firm's production, holding the aggregate inputs fixed. This mechanism is represented by the lower portion of Figure 2, which

¹²When $\psi_n > 0$, Z_{t-1} is a state in period t but not when $\psi_n = 0$, since the value function becomes a static equation.

is similar to the mechanisms in Jaimovich and Floetotto (2008), Bilbiie et al. (2012), and Clementi and Palazzo (2016). It is also supported by Gourio et al. (2016), who show that an increase in net firm entry leads to an increase output and productivity when controlling for economic conditions.

We then substitute for $d \ln Z_t$ using (23) and compare to the model without entry and exit (NE):

$$d \ln Y_t = (1/\vartheta)d \ln a_t + (1 - \alpha)d \ln N_t, \quad (25)$$

$$d \ln Y_t^{NE} = d \ln a_t + \vartheta(1 - \alpha)d \ln N_t. \quad (26)$$

These equations demonstrate that endogenous firm entry and exit cause output to respond more aggressively to changes in technology and aggregate employment. Intuitively, (23) implies that firm entry responds positively to increases in output. Since output is also increasing in the number of active firms by (24), a positive feedback loop amplifies the dynamics of output relative to the case without entry and exit. Quantitatively, (25) shows the amount of amplification due to entry and exit is governed by ϑ , which controls the share of aggregate output attributable to firm profits.

Labor Market Amplification Firm entry and exit is linked to changes in unemployment through the frictional search and matching process. At the center of that market is the vacancy creation condition (20), which describes how vacancies respond to changes in the marginal product of labor, w_t . When w_t increases, the marginal benefit of an employment match increases, employment agencies post more vacancies, and job creation rises. To see this formally, we first differentiate (20) to obtain

$$d \ln q_t = -(1 - \eta)(q_t/\kappa)w_t d \ln w_t, \quad (27)$$

where we abstract from the responses in period $t + 1$ for tractability. To map changes in the job filling rate, q_t , into changes in job creation, M_t , we use the matching function $q_t = 1/(1 + \theta_t^t)^{1/\iota}$ to write $f_t = (1 - q_t^t)^{1/\iota}$ and note that $M_t = f_t U_t^s$. Differentiating these conditions and then substituting into (27) yields the response of job creation to a change in the marginal product of labor,

$$dM_t = (1 - \eta)(M_t/(\kappa\theta_t^t(1 + \theta_t^t)^{1/\iota}))w_t d \ln w_t. \quad (28)$$

To see the effect of entry and exit on w_t , differentiate (11) and then use (25) and (26) to obtain

$$d \ln w_t = (1/\vartheta)d \ln a_t - \alpha d \ln N_t, \quad (29)$$

$$d \ln w_t^{NE} = d \ln a_t - (1 - \vartheta(1 - \alpha))d \ln N_t. \quad (30)$$

Comparing (29) and (30) shows entry and exit amplifies the dynamics of w_t , similar to how it affects output dynamics. Entry and exit strengthen the responses to changes in technology and weaken the offsetting responses to changes in employment. Intuitively, w_t inherits the amplified dynamics of output. Since w_t governs the payoff to vacancy creation, entry and exit amplify the dy-

namics of job creation in (28). These dynamics operate in tandem with the congestion externality in Petrosky-Nadeau and Zhang (2017) and Petrosky-Nadeau et al. (2018). The externality is captured by the denominator in (28), $\theta_t^l(1 + \theta_t^l)^{1/\iota}$, which shows the response of job creation is larger when the labor market is slack and tightness is low. As our quantitative results will show, both sources of nonlinearity are necessary to create realistic skewness in real activity and unemployment dynamics.

3.2 ASYMMETRY An important and intuitive feature of our economy is that firm exit immediately destroys jobs and increases unemployment. In contrast, firm entry does not directly reduce unemployment since new firms hire labor through the frictional search and matching process. The asymmetric effects of entry and exit generate asymmetric and negatively skewed output dynamics.

From (1), the response of the job separation rate to a change in the number of firms is given by

$$ds_t = -(1 - \bar{s})(Z_t/Z_{t-1})\mathbb{I}\{d \ln Z_t < 0\}d \ln Z_t, \quad (31)$$

where the indicator function equals one when firms exit (i.e., when $d \ln Z_t < 0$) and zero otherwise.

Combining (31) with the law of motion for aggregate employment (5) implies

$$d \ln N_t = (1 - \bar{s})(n_{t-1}/n_t)\mathbb{I}\{d \ln Z_t < 0\}d \ln Z_t + dM_t/N_t, \quad (32)$$

where $n_t = N_t/Z_t$ is employment per firm. This expression decomposes the change in employment into changes in job destruction (first term) and job creation (second term). It also shows firm exit causes declines in employment through endogenous separations while firm entry does not directly affect job creation. These mechanisms lead to greater amplification of negative shocks and skewness in employment. They also complement the mechanisms in Ferraro (2018) and Pizzinelli et al. (2020), where negative technology shocks cause endogenous separations at the worker level.

Combining (23), (25), and (32) shows the asymmetric response of aggregate output is given by

$$d \ln Y_t = \frac{1}{1 - (1 - \alpha)(1 - \bar{s})\frac{n_{t-1}}{n_t}\mathbb{I}\{d \ln Z_t < 0\}} \left(\frac{1}{\vartheta} d \ln a_t + (1 - \alpha) \frac{dM_t}{N_t} \right). \quad (33)$$

This result shows that firm exit endogenously strengthens the response of output to changes in technology and new hires, unlike firm entry. Since firm exit only occurs when technology declines, output dynamics are asymmetric. This mechanism is represented by the upper portion of Figure 2. Intuitively, job destruction that occurs when firms exit amplifies the decline in employment and output during recessions. Quantitatively, (33) shows the amount of asymmetry due to entry and exit is decreasing in α , which controls the share of aggregate output attributable to labor.

4 SOLUTION METHOD AND CALIBRATION

This section explains how we solve and calibrate our nonlinear model. In particular, it plots cross-sections of the nonlinear solution and describes the empirical targets that discipline the parameters.

4.1 SOLUTION METHOD AND POLICY FUNCTIONS Endogenous net entry and exit of firms creates nonlinearities that endogenously generate higher-order moments in our model. To accurately capture these effects, we solve the model globally by adapting the policy function iteration algorithm in Richter et al. (2014) to our setting. The algorithm minimizes the Euler equation errors on every node in the discretized state space. It then computes the maximum distance between the policy functions on any node and iterates until that distance falls below a given tolerance criterion.¹³

We make two modifications to the algorithm to account for the inequality constraints. The first is due to entry and exit. Recall that $J_{A,t}^F \in (0, \psi_n)$ when $\Delta Z_t = 0$, $J_{A,t}^F = \psi_n$ when $\Delta Z_t > 0$, and $J_{A,t}^F = 0$ when $\Delta Z_t < 0$. We impose these constraints with an auxiliary variable, $\mu_{A,t}$, that satisfies

$$J_{A,t}^F = \min\{\max\{0, \mu_{A,t}\}, \psi_n\}, \quad \Delta Z_t = \mu_{A,t} - J_{A,t}^F,$$

so $\mu_{A,t} = J_{A,t}^F$ when the value of incumbent firms is between the entry and exit boundaries and the mass of firms is unchanged. At the boundaries, $\mu_{A,t} - J_{A,t}^F$ controls the net entry and exit of firms.

The second modification stems from the constraint on vacancies, $V_t \geq 0$. Following Garcia and Zangwill (1981), we impose this constraint using a second auxiliary variable $\mu_{V,t}$ that satisfies

$$V_t = \max\{0, \mu_{V,t}\}^2, \quad \lambda_{V,t} = \max\{0, -\mu_{V,t}\}^2.$$

$\mu_{V,t}$ maps into vacancies when $V_t > 0$ and the Lagrange multiplier, $\lambda_{V,t}$, when $V_t = 0$. The two conditions are squared to guarantee that they are sufficiently smooth for the algorithm to converge.

To highlight the influence of entry and exit, [Figure 3](#) plots cross-sections of the policy functions for the active firm value, $J_{A,t}^F$, and net entry of active firms, ΔZ_t , as a function of the initial number of active firms and the initial level of technology, which are shown in percent derivations from their steady states. The initial levels of employment and the capital stock are fixed at their steady states.

The nonlinearities in the model are clear. When $\mu_{A,t}$ exceeds the entry cost, $\psi_n = 0.068$, there is net entry of firms to prevent further increases in the value of active firms. This is shown in the upper plateau of the left panel and the upward sloping portion of the right panel. Intuitively, net firm entry is increasing in technology but decreasing in the initial number of active firms. Similarly, when $\mu_{A,t}$ is negative, there is net firm exit to prevent a negative value of active firms, creating the lower plateau and downward sloping portions of the two policy functions. The amount of net firm

¹³We approximate the technology process with an N -state Markov chain following Rouwenhorst (1995) and use piecewise linear interpolation to calculate the period- $t + 1$ policy functions. See [Appendix C](#) for further information.

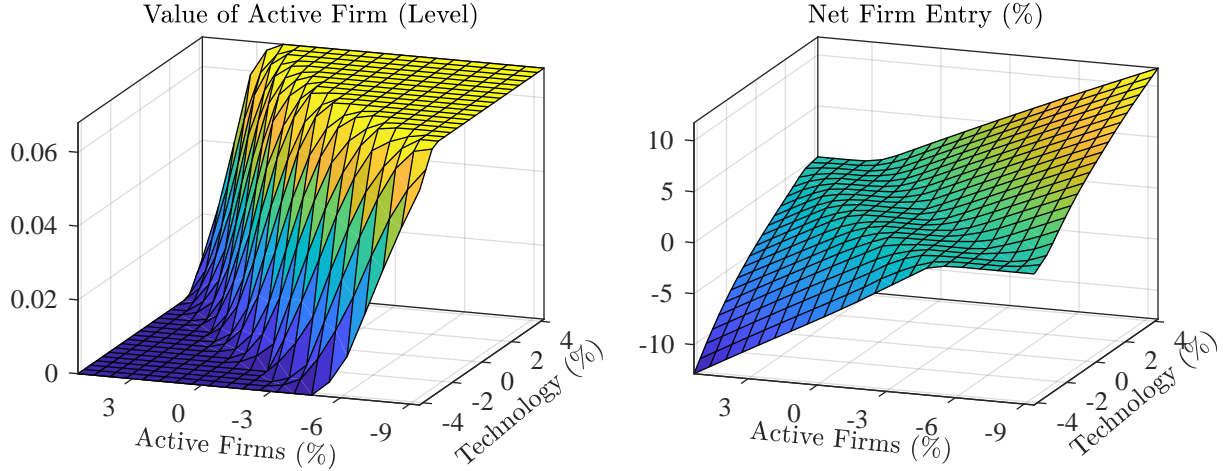


Figure 3: Policy functions for the active firm value and net firm entry. The initial conditions are shown in percent deviations from the deterministic steady state. Employment and capital are fixed at their steady states.

exit is decreasing in technology but increasing in the initial number of active firms. Between 0 and ψ_n , the value of the firm is free to adjust, so there is no change in the number of active firms. This is represented by the upward sloping portion of the left panel and the flat portion of the right panel.

4.2 CALIBRATION The model is calibrated at a monthly frequency to capture employment flows in the data.¹⁴ The discount factor β is set to 0.9983, which implies a 2% average annual real interest rate. We calibrate the rest of the parameters to match data from 1955-2019. The capital depreciation rate, δ , is set to 0.0077 to match the annual average rate on private fixed assets and durable goods. Table 1 summarizes the other parameter values and their corresponding empirical targets.

The parameters governing the returns to scale, ϑ , and the weight on capital in production, α , are set to 0.8656 and 0.2889 to match the average profit and labor shares of income in the non-farm business sector. Lower values of ϑ or α would generate greater amplification and asymmetries, as Section 3 shows. Our value of ϑ is in the middle of the range of micro-estimates of returns to scale. The Fernald (2012) average estimate of 0.33 for the capital share of income maps to an α of 0.2260.

We calibrate χ , κ , and \bar{s} to target the average unemployment rate (5.89%), job finding rate (42.14%), and job separation rate (3.27%). Following Hagedorn and Manovskii (2008), we set η to match the wage-labor productivity elasticity of 0.60. The outside option b governs the economy’s “fundamental surplus fraction” (Ljungqvist and Sargent, 2017), defined as the upper bound on the fraction of a worker’s output that can be allocated to vacancy creation. A small fundamental surplus fraction (i.e., a high b) is crucial to deliver realistic labor market volatility. The matching elasticity, governed by ι , then determines how the volatility is split between vacancies and unemployment (Mortensen and Nagypal, 2007). Therefore, we calibrate b and ι to target the standard

¹⁴Appendix A describes our data sources and how the time series are transformed to construct our empirical targets.

Model Parameter		Value	Empirical Target	Data	Model
Returns to Scale	ϑ	0.8656	Average Profit Share of Income	13.44	13.44
Curvature in Production	α	0.2889	Average Labor Share of Income	61.55	61.55
Intra-Period Search Duration	χ	0.5193	Average Unemployment Rate	5.89	5.93
Vacancy Posting Cost	κ	0.1134	Average Job-Finding Rate	42.15	42.38
Exogenous Separation Rate	\bar{s}	0.0322	Average Job Separation Rate	3.27	3.25
Nash Bargaining Weight	η	0.0997	Wage-Labor Productivity Elasticity*	0.60	0.59
Outside Option	b	0.9702	Unemployment Standard Deviation*	22.28	22.20
Matching Function Curvature	ι	0.6912	Vacancy Standard Deviation*	23.03	22.76
Investment Adjustment Cost	ν	7.1126	Investment Standard Deviation*	8.92	6.22
Firm Entry Cost	ψ_n	0.0680	Entry Share of Job Creation	35.92	35.94
Fixed Production Cost	ψ_y	0.2060	Exit Share of Job Destruction	33.38	33.76
Technology Persistence	ρ_a	0.9473	Output Autocorrelation*	0.91	0.93
Technology Shock SD	σ_a	0.0045	Output Standard Deviation*	3.17	2.67

Table 1: Calibrated parameter values and empirical targets. Monthly time series are averaged to a quarterly frequency. An asterisk denotes a moment based on detrended data, where the empirical trend is computed using a Hamilton (2018) filter with an 8-quarter window and the model trend is equal to the simulated mean.

deviations of detrended unemployment and vacancies, which equal 22.28 and 23.03, respectively.

To discipline the amount of net entry and exit in our model, we target the shares of job creation and job destruction attributable to establishment entry and exit, computed using annual data from the Business Dynamics Statistics database (1978-2018). Empirically, these shares are very stable over time, making them a good calibration target. Over rolling ten year horizons, we find establishment entry on average accounts for 35.92% of total job creation and establishment exit on average accounts for 33.38% of total job destruction.¹⁵ In the model, we first set ψ_y to 0.206, so firm value is normalized to 0 in the deterministic steady state. Given ψ_y and the exit boundary at 0, ψ_n then determines the dynamics of Z_t and the entry and exit shares of job creation and job destruction. To match the shares in the data, we calibrate ψ_n to 0.068, which equals 0.4% of annual output in the deterministic steady state. We validate this choice using several untargeted moments described below.

Finally, we calibrate the autocorrelation of technology, ρ_a , and the shock standard deviation, σ_a , to target the autocorrelation and standard deviation of per capita output. The curvature of the investment adjustment cost function, ν , is set to target the standard deviation of per capita investment. When calibrating these parameters, we also have to account for their effects on other moments, especially the shares of job creation and job destruction due to entry and exit. For example, lowering investment adjustment costs by raising ν makes investment more volatile, which increases employment volatility and the shares of job creation and job destruction attributable to entry and exit. Similarly, increasing the volatility of technology by raising ρ_a or σ_a would increase the volatility of

¹⁵Our calculation of the empirical entry and exit shares is robust to the horizon and consistent with Haltiwanger (2012). Appendix B describes how we compute the shares in the model and provides support for our methodology. It also shows our model is consistent with the amount of net job creation due to the net entry of establishments in the data.

profits and thus the entry and exit shares. Given the focus of our paper, we prioritize matching the entry and exit shares when calibrating the model. Augmenting our model with a richer investment process that can simultaneously match all of our target moments is a useful area for future research.

Moment	Quarterly		Annual	
	BED	Model	BDS	Model
$Corr(Z, U)$	-0.76	-0.91	-0.84	-0.92
$Corr(Z, Y)$	0.67	0.89	0.66	0.91
$Corr(Z, s)$	-0.33	-0.13	-0.05	-0.19
$SD(\Delta \tilde{Z})$	0.35	0.43	1.41	1.05
$Skew(\Delta \tilde{Z})$	-0.58	-0.62	-0.10	-0.25

Table 2: Model validation of entry and exit dynamics. SD , $Skew$, and $Corr$ and denote standard deviation, skewness, and correlation across time. $\Delta \tilde{Z}_t \equiv (Z_t - Z_{t-1}) / ((Z_t + Z_{t-1}) / 2)$ to match the definition of net entry in the data. Monthly values are averaged to a quarterly or annual frequency. The data in levels is detrended, where the empirical trend is computed using a Hamilton (2018) filter with an 8-quarter window and the model trend is equal to the simulated mean. Quarterly establishment data is from the Business Employment Dynamics (BED) database and annual data is from Business Dynamics Statistics (BDS) database.

Validation of Net Entry and Exit Dynamics Our choice to target the shares of job creation and job destruction attributable to establishment entry and exit ensures that the cyclical entry and exit dynamics in our model are the right magnitude relative to the dynamics of employment. To further validate the firm dynamics, Table 2 reports the empirical and model-implied volatility and skewness of net entry, and the correlations between cyclical variation in the number of active firms and a range of outcomes: the unemployment rate, output, and the job separation rate. The empirical moments are computed using establishment level data from the quarterly Business Employment Dynamics database (1992-2019) and annual Business Dynamics Statistics database (1978-2018).

In all cases, the model-implied moments are similar in sign and magnitude to the data. The model reproduces the strong negative correlation between the number of active firms and the unemployment rate (see Figure 1) as well as the positive correlation with output.¹⁶ The model also generates slightly weaker correlations with the job separation rate than the data, suggesting that we do not overstate the strength of the asymmetry mechanism. The dynamics of net entry are also consistent with the empirical evidence. While the model slightly overstates the volatility of net entry at a quarterly frequency, it understates the annual volatility. In both cases, the model generates the negative skewness of net entry driven by the infrequent but large drops in net entry that occur during recessions. Taken together, these results provide support for our calibration approach that exploits the stability of the shares of job creation and job destruction attributable to entry and exit of firms.

¹⁶Crane et al. (2020) show permanent and temporary establishment exit is positively correlated with unemployment.

5 AMPLIFICATION AND ASYMMETRY

With our calibrated model in hand, this section quantifies the key effects of cyclical net entry and exit—amplification of technology shocks and asymmetry in output and unemployment dynamics.

Moment	Data		No Search Frictions		Search Frictions	
	Mean	SE	No Entry/Exit	Entry/Exit	No Entry/Exit	Entry/Exit
$SD(Y)$	3.17	0.26	2.21	2.61	2.20	2.67
$SD(C)$	2.00	0.16	1.57	1.90	1.37	1.71
$SD(I)$	8.92	0.77	5.19	5.96	5.21	6.22
$SD(U)$	22.28	1.85	2.84	3.23	18.01	22.20
$SD(\Delta \ln U)$	5.56	0.57	1.04	1.12	6.17	7.37
$Skew(Y)$	-0.59	0.20	0.01	0.01	-0.31	-0.49
$Skew(C)$	-0.42	0.16	0.00	0.01	-0.30	-0.41
$Skew(I)$	-0.81	0.21	-0.03	-0.03	-0.38	-0.69
$Skew(U)$	0.60	0.20	-0.12	-0.14	0.45	0.64
$Skew(\Delta \ln U)$	1.30	0.26	0.00	-0.01	-0.02	0.51

Table 3: Validation of real activity and unemployment moments. SD and $Skew$ denote standard deviation and skewness across time. The variables in the model are converted to a quarterly frequency by summing the monthly values. The data in levels is reported as a percent deviation from trend. The data trend is based on a Hamilton (2018) filter with an 8-quarter window. The model-implied trend is equal to the simulated mean.

5.1 SIMULATED MOMENTS We begin by comparing simulated moments to their empirical counterparts. We report the standard deviation and skewness of output, consumption, investment, unemployment, and unemployment growth. The skewness in unemployment growth shows whether the model can generate the fast rises and slow declines in the unemployment rate (Neftci, 1984). The model-implied moments are the mean of 20,000 simulations that are initialized with a draw from the ergodic distribution and span 780 months—the same number of observations as our monthly data sample. The simulations are aggregated to a quarterly frequency by averaging across the monthly values to match the frequency of GDP releases. To compute the empirical counterparts, we first detrend the data using a Hamilton (2018) filter with an 8-quarter window. The empirical moments and standard errors (SE) are then estimated using a 2-step Generalized Method of Moments estimator that applies a Newey and West (1987) weighting matrix with 5 lags.

To understand the roles of labor market frictions, entry and exit, and their interaction, we report moments for four versions of the model that remove entry and exit and search frictions in all combinations. When removing search frictions, we model the labor supply using employment lotteries over an indivisible labor choice at the worker level (Hansen, 1985; Rogerson, 1988). To facilitate comparison, we recalibrate the capital adjustment cost and technology parameters so the models without entry and exit generate the same volatilities of output and investment. The entry cost is set

so the two models with entry and exit generate the same shares of job creation and job destruction.¹⁷

Table 3 reports the data and model-implied moments. First consider the models with search and matching frictions. In all cases, the standard deviations are about 20% lower in the model without entry and exit, indicating our benchmark model generates substantial amplification through cyclical input reallocation across firms and the endogenous job destruction that occurs when firms exit. We stress that this amplification occurs under a conservative calibration of the returns to scale parameter, ϑ , which targets the profit share of aggregate income. Lowering ϑ to a commonly chosen value of 0.8 (e.g. Carvalho and Grassi, 2019; Clementi and Palazzo, 2016) would raise the gains from cyclical input reallocation and further strengthen the amplification from firm entry and exit.

The skewness statistics capture asymmetry. Although none of these moments are targeted, our model with entry and exit generates realistic skewness due to the endogenous job destruction. Our baseline model is thus consistent with Sichel (1993), Ferraro (2018), and Dupraz et al. (2019), who document that unemployment exhibits a deepness asymmetry in which its peaks are larger than its troughs. Furthermore, Ferraro (2018) and Pizzinelli et al. (2020) show unemployment skewness is due to job loss, consistent with our mechanism. There is less skewness without entry and exit. In particular, the skewness of output and investment is about 40% smaller and there is no skewness in unemployment growth—a deficiency of search and matching models noted by Ferraro (2017).

Now turn to the models without search and matching frictions. Entry and exit still induces substantial amplification, particularly in real activity. However, the model significantly under-predicts the volatility of unemployment and there is no skewness in any of the variables. These results show that labor market frictions are an important source of volatility and the link between endogenous firm exit and job destruction is an important driver of asymmetry and macroeconomic skewness.¹⁸

5.2 GENERALIZED IMPULSE RESPONSES To further understand the mechanisms that drive the simulated moments, Figure 4 plots generalized impulse responses of output (Y_t), unemployment (U_t), job creation ($M_t = f_t U_t^s$), and job destruction ($s_t N_{t-1}$) to positive and negative technology shocks in our model with a frictional labor market.¹⁹ To demonstrate the amplification and asymmetry, we initialize the simulations at the ergodic mean and consider a shock that is big enough to induce entry and exit. For ease of comparison, we multiply the responses to a positive shock by -1 .

Consider first the responses from the model without entry and exit. The output responses are almost symmetric, while the unemployment responses feature asymmetry that favors negative shocks. This pattern is a consequence of the congestion externality highlighted by Petrosky-Nadeau and Zhang (2017) and Petrosky-Nadeau et al. (2018). For example, when technology de-

¹⁷Appendix D shows the nonlinear equilibrium system for the indivisible labor model and the calibrated parameters.

¹⁸These results also show that capital adjustment costs are not an independent source of macroeconomic skewness.

¹⁹Following Koop et al. (1996), the response of x_{t+h} over horizon h is given by $\mathcal{G}_t(x_{t+h}|\varepsilon_{a,t+1} = 2SD(\hat{a}), \mathbf{z}_t) = E_t[x_{t+h}|\varepsilon_{a,t+1} = 2SD(\hat{a}), \mathbf{z}_t] - E_t[x_{t+h}|\mathbf{z}_t]$, where \mathbf{z}_t is a vector of initial states and $2SD(\hat{a})$ is the size of the shock.

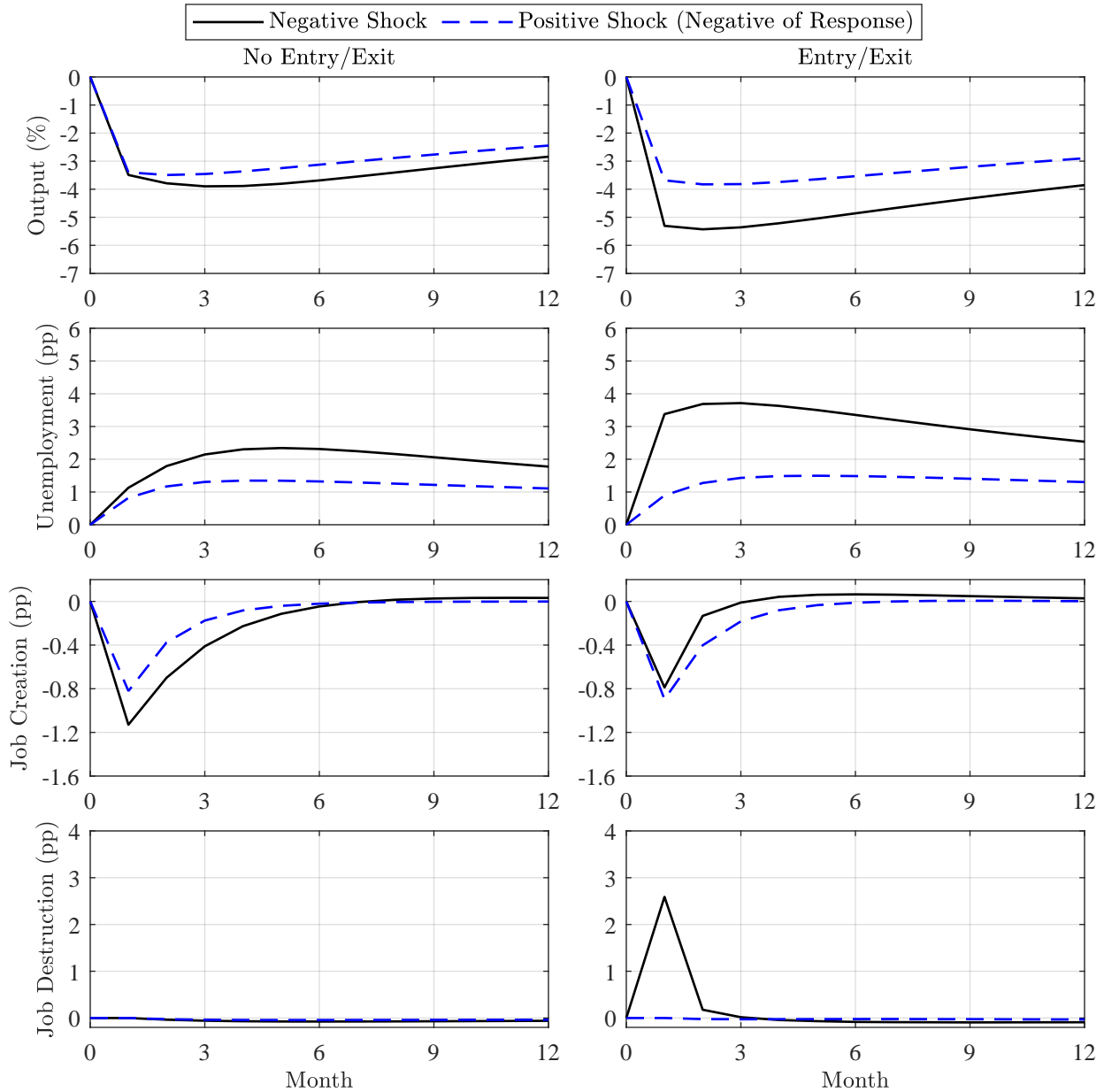


Figure 4: Generalized impulse responses to a technology shock in normal times, $(U_0, Z_0) = (6\%, 1)$.

clines the marginal benefit of vacancy creation falls, so employment agencies post fewer vacancies and unemployment rises. This increases the ratio of unemployed searching to vacancies, which worsens labor market congestion and increases the sensitivity of the job finding rate to further declines in vacancy postings. As a result, vacancies and hence unemployment responds more to negative technology shocks than positive shocks. Despite this mechanism, the model without firm entry and exit does not generate much asymmetry in output because firms do not exit when profits fall.

Now turn to the model with entry and exit. The asymmetry mechanism is immediately visible.

In response to a negative technology shock, firm exit and the ensuing job destruction causes unemployment to spike on impact rather than increase gradually. This fast rise followed by a slow decline is the source of the skewness in unemployment growth shown in [Table 3](#). The combination of the asymmetry and amplification mechanisms cause output to respond noticeably more to the negative shock, and also to inherit the fast-slow dynamic of unemployment. Interestingly, job creation has very little role, as the responses are symmetric and similar to the model without entry and exit.

State-Dependence We also use impulse responses to study how the amplification and asymmetry mechanisms vary with the state of the economy. To show this state-dependence, [Figure 5](#) plots the same responses as [Figure 4](#) when the economy is initially in a recession ($Z_0 = 0.97$; $U_0 = 8\%$). The economy becomes more sensitive to technology shocks in both models, but the amplification is stronger in our benchmark model. Furthermore, the model without entry and exit continues to exhibit relatively little asymmetry, while our benchmark model responds even more strongly to the negative shock. This result is consistent with Pizzinelli et al. (2020), who show empirically that productivity shocks have a larger effect on the unemployment and job separation rates in periods of low productivity than in periods of high productivity. Intuitively, when a negative shock hits during a recession in our benchmark model, firms' already low profit margins decline even more leading to further net exit of firms and additional job destruction, which exacerbates the recession. Thus, the distribution of future output growth is more negatively skewed starting from a recessionary state, which has important consequences for macroeconomic uncertainty that we describe below.²⁰

5.3 ERGODIC DISTRIBUTIONS Standard deviation and skewness are useful summary statistics to quantify the key mechanisms in the model, but they mask distributional differences between the models with and without firm entry and exit. To see these differences, [Figure 6](#) plots 2,000 draws from the ergodic distributions of the variables shown in [Figures 4](#) and [5](#) as a function of technology.

First consider the output distributions. The range of possible output realizations at a given technology level is wider in the model with entry and exit, in line with the higher standard deviation reported in [Table 3](#). Furthermore, the differences in the range of outcomes between the two models is larger when technology is lower, meaning that declines in technology can cause larger declines in output when firms endogenously exit and destroy jobs. For example, a 1% decline in technology can cause output to fall by as much as 7% in the benchmark model, but by no more than 5% in the model without cyclical net entry and exit. These differences are the source of the higher skewness.

Similar results hold for the unemployment distributions. The range of unemployment outcomes at a given technology level is wider with entry and exit due to the amplification mechanism, and the range becomes relatively wider as technology declines, creating positive skewness. The distri-

²⁰[Appendix E](#) compares the impulse responses to two different shock sizes. Once again, there is significant nonlinearity. Doubling the shock size more than doubles the responses. The nonlinearity occurs because the two shocks lead to different amounts of endogenous firm exit and job destruction, just like we show for different states of the economy.

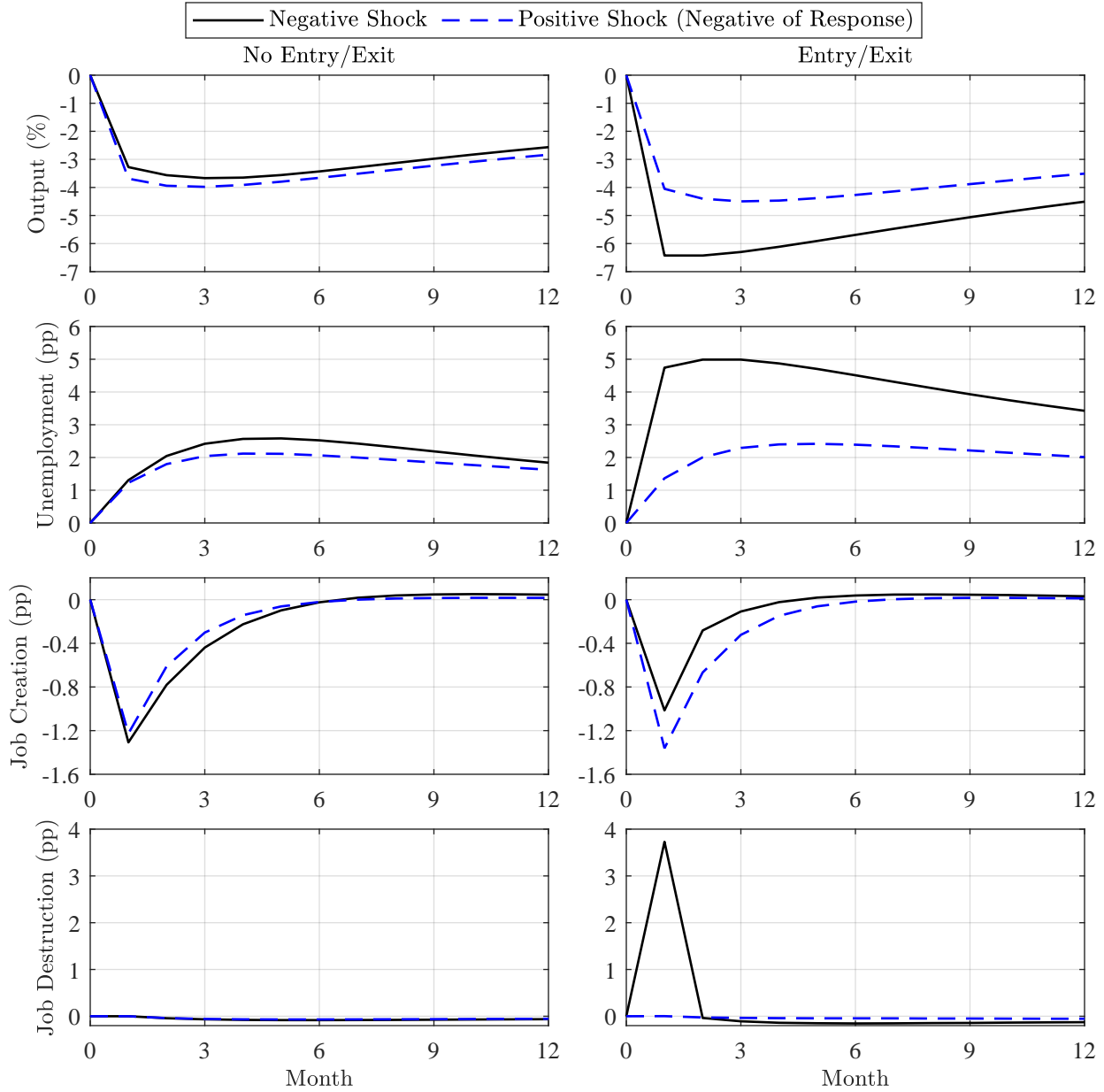


Figure 5: Generalized impulse responses to a technology shock in a recession, $(U_0, Z_0) = (8\%, 0.97)$.

Contributions of job creation are very similar across the two models. The differences between the models are driven by the increase in job destruction that occurs when firms exit in low technology states.

6 IMPLICATIONS OF NON-GAUSSIAN DYNAMICS

We have shown that our baseline model with entry and exit amplifies and skews the state-dependent transmission of technology shocks. This section emphasizes two implications of our results: a higher welfare cost of business cycles and empirically consistent time-varying output uncertainty.

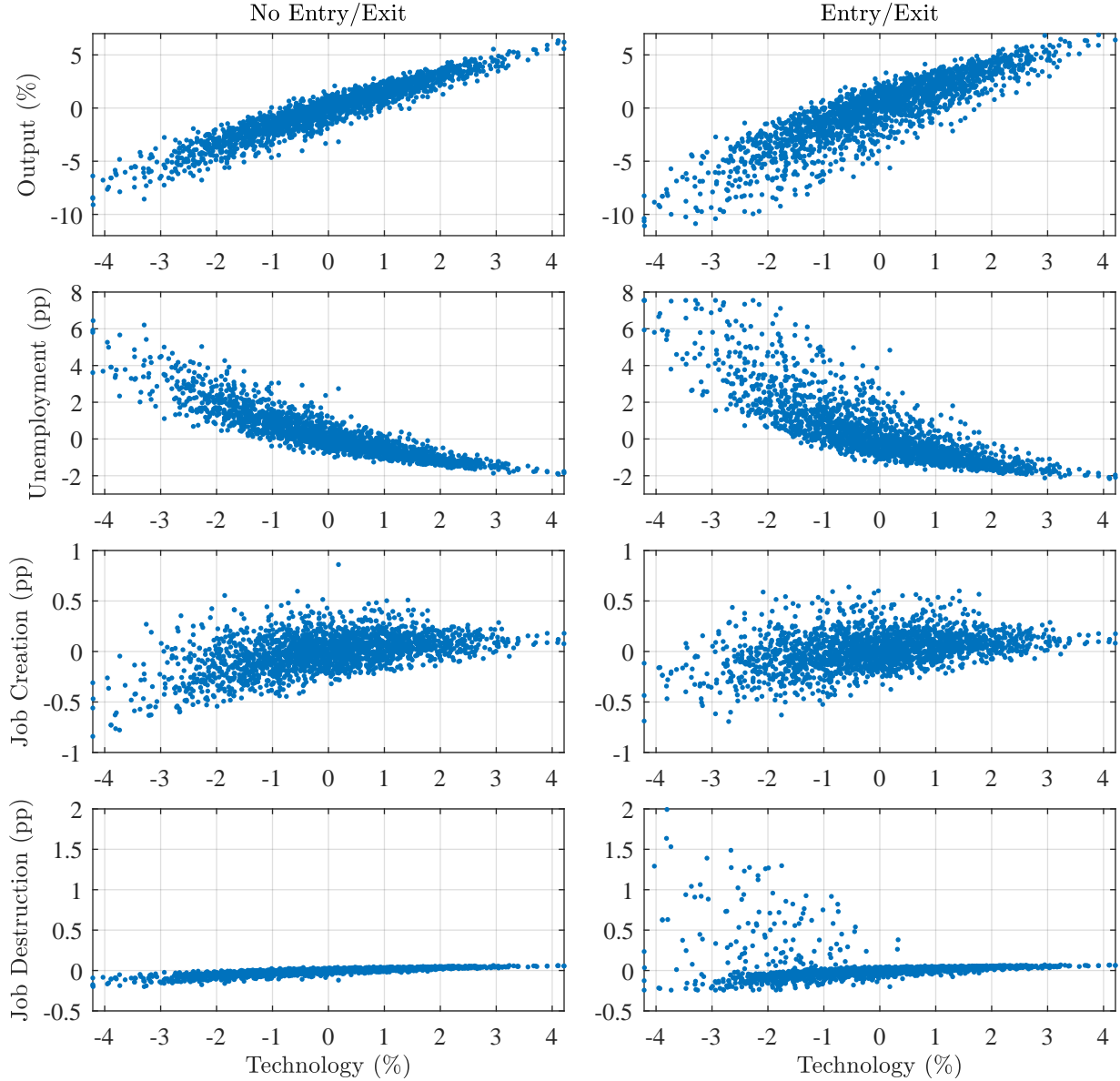


Figure 6: Ergodic distributions as a function of technology.

6.1 WELFARE COST OF BUSINESS CYCLES To compute the welfare cost of business cycles, we implement the experiment in Lucas (1987, 2003). First, we compute the representative household's lifetime utility in an economy without shocks in which consumption always equals its stochastic steady state, \tilde{C} . Second, we compute expected welfare in the stochastic economy. Finally, we compute the percentage of stochastic consumption λ households would require to make them indifferent between the two consumption paths. Formally, we compute $\lambda = 100 \times (\exp(\ln \tilde{C} - \frac{1-\beta}{1-\beta^{T-1}} \frac{1}{N_E} \sum_{j=1}^{N_E} E_0[\sum_{t=0}^T \beta^t \ln C_{j,t} | \mathbf{z}_{j,0}]) - 1)$, where $T = 3000$, $\mathbf{z}_{j,0}$ is a

draw from the ergodic distribution with consumption path $\{C_{j,t}\}$, and N_E is the number of draws.

We find $\lambda = 0.42\%$, so that households require an additional 0.42% of consumption in each period to accept the fluctuations from business cycles in our model economy with endogenous entry and exit of firms. This number is over 8 times larger than the cost reported in Lucas (2003, $\lambda = 0.05\%$), which assumes that aggregate consumption is a Gaussian process. It also 1.5 times larger than the cost of business cycles in the economy with only search frictions ($\lambda = 0.27\%$), indicating that there is a significant cost to the endogenous net entry and exit of firms over the business cycle.²¹

Relative to the literature, we do not rely on extreme risk aversion, non-standard utility functions, or household heterogeneity to generate the larger cost of business cycles. Instead, the cost is driven by the extra negative skewness that cyclical entry and exit imparts to consumption, over and above the skewness generated by search and matching frictions. To see this, we can compute a third-order approximation of the average utility loss around the stochastic steady state, given by

$$E[\ln \tilde{C} - \ln C_t] \approx \frac{1}{2}SD(\ln C_t)^2 - \frac{1}{3}SKEW(\ln C_t)SD(\ln C_t)^3.$$

This shows the average utility loss increases as consumption becomes more negatively skewed. As a back-of-the-envelope calculation, we can set $SD(\ln C_t) = 1.71$ from Table 3, and then compute the increase in utility loss due to the 0.1 decline in skewness caused by cyclical net entry and exit:

$$\Delta E[\ln \tilde{C} - \ln C_t] \approx \frac{1}{3}(0.1)SD(\ln C_t)^3 = 0.164,$$

which roughly coincides with the increase in the welfare cost when we add cyclical entry and exit to the model with only search frictions (i.e., λ increases from 0.27% to 0.42%, an increase of 0.15%).

6.2 ENDOGENOUS UNCERTAINTY Recent empirical work finds that macroeconomic uncertainty increases in recessions, and it is often an endogenous response to exogenous first moment shocks to output (Ludvigson et al., 2020). This contrasts with a large literature that models uncertainty as exogenous volatility shocks (e.g., Bloom, 2009; Fernández-Villaverde et al., 2015, 2011; Justiniano and Primiceri, 2008; Leduc and Liu, 2016). Our model is consistent with this causal relationship, qualitatively and quantitatively. To demonstrate this, we follow Jurado et al. (2015) and define uncertainty as the conditional volatility of the forecast error in h -period ahead output:

$$\mathcal{U}_{t,t+h}^Y = \sqrt{E_t[(\Delta \ln Y_{t+h} - E_t[\Delta \ln Y_{t+h}])^2]}.$$

We measure uncertainty in the data as the quarterly average of the monthly real uncertainty series ($h = 1$) from Ludvigson et al. (2020). This series is a sub-index of the macro uncertainty series from Jurado et al. (2015) that accounts for 73 real activity variables. Repeated simulations of a fac-

²¹The welfare cost without firm entry and exit is consistent with Hairault et al. (2010) and Jung and Kuester (2011). Relatedly, Pizzinelli et al. (2018) show that skewness in the unemployment rate creates state-dependent welfare costs.

tor augmented vector autoregression are used to obtain estimates of uncertainty for each real variable and then averaged to generate the aggregate real uncertainty series. Before estimating, most variables are transformed into growth rates and standard normalized. To make the units from our model comparable to the real uncertainty series, we define $SD(\mathcal{U}^Y) \equiv SD(\mathcal{U}_{t,t+1}^Y)/SD(\Delta \ln Y_t)$.

Moment	Data		Model	
	Mean	SE	No Entry/Exit	Entry/Exit
$SD(\mathcal{U}^Y)$	5.68	0.51	3.10	6.94
$AC(\mathcal{U}^Y)$	0.84	0.04	0.93	0.78
$Corr(\Delta \ln Y, \mathcal{U}^Y)$	-0.37	0.09	-0.17	-0.42

Table 4: Macroeconomic uncertainty moments. SD , AC , and $Corr$ denote standard deviation, autocorrelation, and cross-correlation across time. Output in the model is converted to a quarterly frequency by summing the monthly values. $SD(\mathcal{U}^Y)$ is normalized by $SD(\Delta \ln Y)$ to match the units of uncertainty data.

Table 4 compares model-implied uncertainty moments to their counterparts in the data. Empirically, uncertainty is volatile, persistent, and strongly counter-cyclical. Our baseline model endogenously matches these features without the aid of exogenous volatility shocks. In the model, fluctuations in uncertainty are endogenous responses to first moment technology shocks, consistent with Ludvigson et al. (2020). Intuitively, the state of the economy affects the probability of firm exit and therefore the shock transmission. Realistic state-dependence, captured by time-varying uncertainty, is an essential feature of a successful business cycle model. Without net entry and exit, the volatility of uncertainty significantly undershoots the data and is less counter-cyclical, suggesting that search and matching frictions alone are insufficient to generate realistic uncertainty dynamics.

7 CONCLUSION

This paper studies how cyclical net entry and exit of firms affects business cycle dynamics. We show that entry and exit amplifies and skews the transmission of technology shocks. Quantitatively, we find that extending a canonical macroeconomic model to account for the interaction of entry and exit and labor market frictions improves its ability to generate realistic business cycle dynamics, including the asymmetric nature of recessions versus expansions. These results offer a new lens through which policymakers can respond to cyclical changes in the economy.²² They also complement a large literature that studies the effects of long-run changes in the distribution of firms. Integrating our insights into that growing class of models is an exciting avenue for future research.

²²Cacciatore and Fiori (2016) and Bilbiie et al. (2019) are the first to study optimal policy in this class of models.

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A DATA SOURCES AND TRANSFORMATIONS

We use the following time-series from 1955-2019 provided by Haver Analytics:

1. **Civilian Noninstitutional Population: 16 Years and Over**,
Not Seasonally Adjusted, Quarterly, Thousands (LN16N@USECON)
2. **Gross Domestic Product: Implicit Price Deflator**,
Seasonally Adjusted, Quarterly, 2012=100 (DGDP@USNA)
3. **Gross Domestic Product**,
Seasonally Adjusted, Quarterly, Billions of Dollars (GDP@USECON)
4. **Personal Consumption Expenditures: Nondurable Goods**,
Seasonally Adjusted, Quarterly, Billions of Dollars (CN@USECON)
5. **Personal Consumption Expenditures: Durable Goods**,
Seasonally Adjusted, Quarterly, Billions of Dollars (CD@USECON)
6. **Personal Consumption Expenditures: Services**,
Seasonally Adjusted, Quarterly, Billions of Dollars (CS@USECON)
7. **Private Fixed Investment**,
Seasonally Adjusted, Quarterly, Billions of Dollars (F@USECON)
8. **Unemployed**, Seasonally Adjusted, Monthly, Thousands, 16 years+ (LTU@USECON)
9. **Labor Force**, Seasonally Adjusted, Monthly, Thousands, 16 years+ (LF@USECON)
10. **Unemployed Less Than 5 Weeks**,
Seasonally Adjusted, Monthly, Thousands, 16 years and over (LU0@USECON)
11. **Job Separation Rate**, Job Openings and Labor Turnover Survey,
Seasonally Adjusted, Monthly, Percent of Employment (LJSTPA@USECON)
12. **Job Openings**, Job Openings and Labor Turnover Survey,
Seasonally Adjusted, Monthly, Thousands (LJJTLA@USECON)
13. **Output Per Person**, Non-farm Business Sector, All Persons,
Seasonally Adjusted, Quarterly, 2012=100 (LXNFS@USNA)
14. **Labor Share**, Non-farm Business Sector, All Persons,
Seasonally Adjusted, Percent (LXNFBL@USNA)
15. **Nonfarm Proprietors' Income with IVA & CCA_{adj}**,
Seasonally Adjusted, Quarterly, Billions of Dollars (YOPN@USNA)

16. **Corporate Profits After Tax with IVA & CCAdj**,
Seasonally Adjusted, Quarterly, Billions of Dollars (YCATJ@USNA)
17. **Private Sector Opening Establishments**,
Seasonally Adjusted, Quarterly, % of Total Establishments (LRJEGO@USECON)
18. **Private Sector Closing Establishments**,
Seasonally Adjusted, Quarterly, % of Total Establishments (LRJELC@USECON)
19. **Net Stock: Private Fixed Assets**, Billions of Dollars, Annual (EPT@CAPSTOCK)
20. **Net Stock: Consumer Durable Goods**, Billions of Dollars, Annual (EDT@CAPSTOCK)
21. **Depreciation: Private Fixed Assets**, Billions of Dollars, Annual (KPT@CAPSTOCK)
22. **Depreciation: Consumer Durable Goods**, Billions of Dollars, Annual (KDT@CAPSTOCK)

We also used the following data from other sources:

1. **Help Wanted Advertising Index (HWI)**, based on Barnichon (2010) and in units of the labor force. The series corrects for online advertising and is available on the author's [website](#).
2. **Real Uncertainty (U)**, 1-quarter horizon, based on Ludvigson et al. (2020). The series is available on Ludvigson's [website](#). The monthly series is averaged to a quarterly frequency.
3. **Business Dynamics Statistics**, 2018 database, published by the [Census Bureau](#).
 - **Establishments (E)**, Count of the number of establishments.
 - **Establishment Entry (EN)**, Count of establishments born.
 - **Establishment Exit (EX)**, Count of establishments exiting.
 - **Job Creation (JC)**, employment gains from expanding and opening establishments.
 - **Job Creation Births (JCB)**, employment gains from establishment openings.
 - **Job Destruction (JD)**, employment losses from shrinking and closing establishments.
 - **Job Creation Deaths (JCD)**, employment losses from establishment closings.

We applied the following transformations to the above data sources:

1. **Per Capita Real Output Growth:**

$$\Delta \ln Y_t = 100 \left(\ln \left(\frac{GDP_t}{DGD P_t \times LN16N_t} \right) - \ln \left(\frac{GDP_{t-1}}{DGD P_{t-1} \times LN16N_{t-1}} \right) \right).$$

2. **Per Capita Real Consumption Growth:**

$$\Delta \ln C_t = 100 \left(\ln \left(\frac{CN_t + CS_t}{DGD P_t \times LN16N_t} \right) - \ln \left(\frac{CN_{t-1} + CS_{t-1}}{DGD P_{t-1} \times LN16N_{t-1}} \right) \right).$$

3. Per Capita Real Investment Growth:

$$\Delta \ln I_t = 100 \left(\ln \left(\frac{F_t + CD_t}{DGD P_t \times LN16N_t} \right) - \ln \left(\frac{F_{t-1} + CD_{t-1}}{DGD P_{t-1} \times LN16N_{t-1}} \right) \right).$$

4. **Unemployment Rate:** $U_t = 100(LTU_t/LF_t)$.

5. **Vacancy Rate:** HWI from 1955M1-2000M12 and $LJJTLA/LF$ from 2001M1-2019M12.

6. **Short-term Unemployed (U^s):** The redesign of the Current Population Survey (CPS) in 1994 reduced u_t^s . To correct for this bias, we use IMPUMS-CPS data to scale u_t^s by the ratio of u_t^s/u_t for the first and fifth rotations groups to u_t^s/u_t across all rotation groups. In addition to the 9 mandatory identification variables, we first extract the following: EMPSTAT (“Employment Status”), DURUNEMP (“Continuous weeks unemployed”) and MISH (“Month in sample, household level”). Unemployed persons have EMPSTAT of 20, 21, or 22. Short-term unemployed are persons who are unemployed and DURUNEMP is 5 or less. Incoming rotation groups have MISH of 1 or 5. Using the final weights, WTFINL, we then calculate unemployment rates conditional on the appropriate values of MISH and DURUNEMP. We then apply the X-12 seasonal adjustment function in STATA to the time series for the ratio. Finally, we take an average of the seasonally adjusted time series. This process yields an average ratio of 1.1693, so U^s equals $LU0$ before 1994 and $1.1693 \times LU0$ after 1994.

7. **Job-Finding Rate:** $f_t = 100(LTU_t - U_t^s)/LTU_t$.

8. **Real Wage:** $w_t = L X N F B L_t \times L X N F S_t$

9. **Wage Elasticity:** Slope coefficient from regressing $\ln w_t$ on an intercept and $\ln L X N F S_t$.

10. **Profit Share of Income:** $D_t/Y_t = (YOPN_t + YCATJ_t)/GDP_t$

11. **Net Entry:** $\Delta \tilde{Z}_t \equiv LRJEGO_t - LRJELC_t$ or $\Delta \tilde{Z}_t \equiv (EN_t - EX_t)/((E_t + E_{t-1})/2)$

12. **Establishments:** $LRJEGO_t$ and $LRJELC_t$ are reported as a percent of the 2-quarter moving average of total establishments, so $\Delta \tilde{Z}_t = (Z_t - Z_{t-1})/((Z_t + Z_{t-1})/2)$ where Z_t is total establishments. Let $Z_0 = 1$, and recursively update $Z_t = (2 + \Delta Z_t)Z_{t-1}/(2 - \Delta Z_t)$.

13. **Depreciation Rate:** $\delta = (1 + \frac{1}{T} \sum_{t=1}^T (KPT_t + KDT_t)/(EPT_{t-1} + EDT_{t-1}))^{1/12} - 1$.

14. **Entry Share of Job Creation:** $\omega^{JC} = \frac{100}{T-H+1} \sum_{t=H}^T \sum_{j=t-H+1}^t JCB_j / \sum_{j=t-H+1}^t JC_j$.

15. **Exit Share of Job Creation:** $\omega^{JD} = \frac{100}{T-H+1} \sum_{t=H}^T \sum_{j=t-H+1}^t JCD_j / \sum_{j=t-H+1}^t JC_j$.

B ENTRY AND EXIT SHARES OF JOB CREATION AND JOB DESTRUCTION

This section explains how we map firm dynamics in the model to the data. We first compute total job creation and job destruction by aggregating the monthly flows over rolling ten year windows:

$$\begin{aligned} JC_{j,j+H} &= \sum_{t=0}^{H-1} Z_{j+t-1} \max\{0, n_{j+t} - n_{j+t-1}\} + \sum_{t=0}^{H-1} \max\{0, Z_{j+t} - Z_{j+t-1}\} n_{j+t}, \\ JD_{j,j+H} &= \sum_{t=0}^{H-1} Z_{j+t-1} \max\{0, n_{j+t-1} - n_{j+t}\} + \sum_{t=0}^{H-1} \max\{0, Z_{j+t-1} - Z_{j+t}\} n_{j+t}, \end{aligned}$$

where $n_t = N_t/Z_t$ is the mass of workers employed by each firm in month t . $JC_{j,j+H}$ is the job creation from month j to month $j+H$ due to the expansion of existing establishments (first term) and entering establishments (second term). Analogously, $JD_{j,j+H}$ is the job destruction from month j to month $j+H$ due to the contraction of existing establishments and exiting establishments. The shares of job creation and job destruction attributable to establishment entry and exit are given by

$$\omega_{j,j+H}^{JC} = \frac{\sum_{t=0}^{H-1} \max\{0, Z_{j+t} - Z_{j+t-1}\} n_{j+t}}{JC_{j,j+H}}, \quad \omega_{j,j+H}^{JD} = \frac{\sum_{t=0}^{H-1} \max\{0, Z_{j+t-1} - Z_{j+t}\} n_{j+t}}{JD_{j,j+H}}.$$

The shares in our model are based on cyclical variation in job flows, while the empirical shares contain cyclical and trend components. Fortunately, the stability of the empirical shares provides evidence that the cyclical component shares equal the trend component shares. To see this, break total job creation JC and job creation due to firm births JCB into trend and cyclical components:

$$\begin{aligned} JC_{j,j+H} &= JC_{j,j+H}^T + JC_{j,j+H}^C, \\ JCB_{j,j+H} &= JCB_{j,j+H}^T + JCB_{j,j+H}^C. \end{aligned}$$

Then the empirical share can be written as a weighted average of the trend and cyclical components:

$$\omega_{j,j+H} = \frac{JCB_{j,j+H}^T + JCB_{j,j+H}^C}{JC_{j,j+H}^T + JC_{j,j+H}^C} = \left(1 - \frac{JC_{j,j+H}^C}{JC_{j,j+H}^T + JC_{j,j+H}^C}\right) \frac{JCB_{j,j+H}^T}{JC_{j,j+H}^T} + \frac{JC_{j,j+H}^C}{JC_{j,j+H}^T + JC_{j,j+H}^C} \frac{JCB_{j,j+H}^C}{JC_{j,j+H}^C}.$$

Stability of the empirical share implies that the trend share is constant, $\omega_H^T = \frac{JCB_{j,j+H}^T}{JC_{j,j+H}^T}$, and approximately equal to the empirical share, $\omega_{j,j+H} \approx \omega_H^T$. Imposing these conditions then implies that the cyclical share must also approximately equal the trend share, $\omega_H^C \approx \frac{JCB_{j,j+H}^C}{JC_{j,j+H}^C}$. Therefore, we can use the average empirical shares as targets to discipline the cyclical shares generated by the model.

To construct total job creation and job destruction in the model, we aggregate monthly net flows into H -month gross flows. As a validity check for this approach, we also construct net flows directly in the data using the model as a guide. Using filtered data on establishments, Z_t , and establishment size, n_t , we compute annual net job creation by the net entry of establishments in year t :

$$NJC_t^{EE} = (Z_t - Z_{t-1})n_t.$$

Using the annual data from the Business Dynamics Statistics database, the standard deviation of this series is 1.37%. When we compute the same statistic on model-generated data, we obtain a standard deviation of 0.96%. Hence, calibrating the model based on the aggregation of net flows into gross flows actually slightly understates the volatility of net flows in the data. This provides further evidence that our calibration of the entry cost does not overstate the role of entry and exit.

C SOLUTION METHOD

The equilibrium system is given by $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1}) | \mathbf{z}_t, \Theta] = 0$, where g is a vector-valued function, \mathbf{x}_t is a vector of variables, ε_t is a vector of shocks, \mathbf{z}_t is a vector of states, and Θ is a vector of parameters. The state vector consists of technology, employment, the capital stock, and active firms, $\mathbf{z}_t = [a_t, N_{t-1}, Z_{t-1}, K_{t-1}]$. We discretize a_t , N_{t-1} , Z_{t-1} , and K_{t-1} into 10, 15, 15, and 15 evenly-spaced points, respectively. The bounds on the three endogenous state variables, N_{t-1} , Z_{t-1} , and K_{t-1} , are set to $[-8.0\%, +2.5\%]$, $[-9.0\%, +5.5\%]$, and $[-7.0\%, +7.0\%]$ of their deterministic steady-state values. Those bounds contain at least 99% of their ergodic distributions.

There are many ways to discretize the exogenous state, a_t . We use the Markov chain in Rouwenhorst (1995), which Kopecky and Suen (2010) show outperforms other methods for approximating autoregressive processes. The realization of \mathbf{z}_t on node d is denoted $\mathbf{z}_t(d)$. This method provides integration nodes, $[a_{t+1}(m)]$, with weights, $\phi(m)$, for $m \in \{1, \dots, M\}$. Since technology follows a Markov chain, the realizations of a_{t+1} are the exact same as a_t ($M = 10$).

Vacancies are subject to a nonnegativity constraint, $V_t \geq 0$. To impose the constraint, we introduce an auxiliary variable, $\mu_{V,t}$, such that $V_t = \max\{0, \mu_{V,t}\}^2$ and $\lambda_{V,t} = \max\{0, -\mu_{V,t}\}^2$, where $\lambda_{V,t}$ is the Lagrange multiplier on the non-negativity constraint. If $\mu_{V,t} \geq 0$, then $V_t = \mu_{V,t}^2$ and $\lambda_{V,t} = 0$. When $\mu_{V,t} < 0$, the constraint is binding, $V_t = 0$, and $\lambda_{V,t} = \mu_{V,t}^2$. Therefore, the vacancy constraint is transformed into a pair of equalities following Garcia and Zangwill (1981).

There is also an inequality constraint on the value function of active firms, $\psi_x \leq J_{A,t}^F < \psi_n$, where ψ_x is a general scrap value (we set $\psi_x = 0$ in the final solution). To impose this inequality constraint, we create a second auxiliary variable, $\mu_{A,t}$, that equals $J_{A,t}^F$ when there is no entry or exit and the change in active firms, $\lambda_{A,t} = Z_t - Z_{t-1}$, when entry or exit occurs. When firms enter $J_{A,t}^F = \psi_n$ and when they exit $J_{A,t}^F = \psi_x$. Therefore, the state variable for active firms, Z_{t-1} , is updated according to $J_{A,t}^F = \max\{\psi_n, \min\{\psi_x, \mu_{A,t}\}\}$, $\lambda_{A,t} = \mu_{A,t} - J_{A,t}^F$, and $Z_t = Z_{t-1} + \lambda_{A,t}$. Therefore, the fraction of incumbent firms who remain active is given by $\xi_t = 1 + \frac{\lambda_{F,t}}{Z_{t-1}} \mathbb{I}(\lambda_{F,t} < 0)$.

The vector of policy functions and the realization on node d are denoted $\mathbf{p}\mathbf{f}_t$ and $\mathbf{p}\mathbf{f}_t(d)$, where $\mathbf{p}\mathbf{f}_t \equiv [\mu_{V,t}(\mathbf{z}_t), \mu_{A,t}(\mathbf{z}_t)]$. The following steps outline our policy function iteration algorithm:

1. Use Sims's (2002) `gensys` algorithm to solve the log-linear model without entry and exit and obtain an initial conjecture for $\mu_{V,t}(d)$. Guess that $\mu_{A,t}(d) = 0.5(\psi_n + \psi_x)$ for all $d \in D$.

2. Solve the nonlinear model without entry and exit by setting $\psi_n = 100$ and $\psi_x = -100$.
- (a) On iteration $j \in \{1, \dots\}$ and each node $d \in \{1, \dots, D\}$, use Chris Sims's `csolve` to find $\mathbf{pf}_t(d)$ to satisfy $E[g(\cdot)|\mathbf{z}_t(d), \Theta] \approx 0$. Guess $\mathbf{pf}_t(d) = \mathbf{pf}_{j-1}(d)$ and implement:
- i. Solve for all variables dated at time t , given $\mathbf{pf}_t(d)$ and $\mathbf{z}_t(d)$.
 - ii. Linearly interpolate the policy functions, \mathbf{pf}_{j-1} , at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mathbf{pf}_{t+1}(m)$ on every integration node, $m \in \{1, \dots, M\}$.
 - iii. Given $\{\mathbf{pf}_{t+1}(m)\}_{m=1}^M$, solve for the other elements of $\mathbf{x}_{t+1}(m)$, noting that ξ_{t+1} , defined in (15), depends on the fraction of firms that exit at $t + 1$. Then compute:

$$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1})|\mathbf{z}_t(d), \Theta] \approx \sum_{m=1}^M \phi(m)g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$
 - iv. When `csolve` converges, set $\mathbf{pf}_j(d) = \mathbf{pf}_t(d)$.
- (b) Repeat step 2 until $\text{maxdist}_j < 10^{-8}$, where $\text{maxdist}_j \equiv \max\{|\mathbf{pf}_j - \mathbf{pf}_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate solution.
3. Solve the baseline nonlinear model by setting $s_t = \bar{s} + (1 - \bar{s})(1 - \xi_t)\zeta$, where $\zeta \in [0, 1]$. Using the solution without entry and exit as an initial conjecture, iterate from $\zeta = 0.25$ to $\zeta = 1$, each time solving the model using the previous solution as an new initial conjecture.

The algorithm is programmed in Fortran 90 with Open MPI and run on the BigTex supercomputer.

D INDIVISIBLE LABOR MODEL

Relative to the baseline model, we remove search and matching frictions but extend the representative household's preferences to include linear disutility of labor. The equilibrium system is given by

$$\begin{aligned}
 J_{A,t}^F &= \min\{\max\{\psi_x, \mu_{F,t}\}, \psi_n\}, \\
 J_{X,t}^F &= \max\{\psi_x, J_{A,t}^F\}, \\
 \lambda_{F,t} &= \mu_{F,t} - J_{A,t}^F, \\
 w_t &= C_t, \\
 Y_t &= a_t Z_t^{1-\vartheta} (K_{t-1}^\alpha N_t^{1-\alpha})^\vartheta, \\
 C_t + I_t &= Y_t, \\
 K_t &= (1 - \delta)K_{t-1} + \left(a_1 + \frac{a_2}{1-1/\nu} \left(\frac{I_t}{K_{t-1}} \right)^{1-1/\nu} \right) K_{t-1}, \\
 \frac{1}{a_2} \left(\frac{X_t}{K_{t-1}} \right)^{1/\nu} &= E_t \left[x_{t+1} \left(\alpha \vartheta \frac{Y_{t+1}}{K_t} + \frac{1}{a_2} \left(\frac{I_{t+1}}{K_t} \right)^{1/\nu} (1 - \delta + a_1) + \frac{1}{\nu-1} \frac{I_{t+1}}{K_t} \right) \right], \\
 w_t &= (1 - \alpha)\vartheta Y_t / N_t, \\
 Z_t &= Z_{t-1} + \lambda_{F,t}, \\
 Z_t J_{A,t}^F &= (1 - \vartheta)Y_t - Z_t \psi_y + Z_t E_t[x_{t+1} J_{X,t+1}^F], \\
 \ln a_{t+1} &= (1 - \rho_a) \ln \bar{a} + \rho_a \ln a_t + \sigma_a \varepsilon_{a,t+1}.
 \end{aligned}$$

We recalibrate $\nu = 9.879$, $\rho_a = 0.983$, and $\sigma_a = 0.003$ to match the standard deviation of investment and the autocorrelation and standard deviation of output in the search and matching model without entry and exit. We also set $\psi_n = 0.0365$ to match the entry and exit shares of job creation and job destruction in our baseline model. All of the other parameters in [Table 1](#) are unchanged.

Solution We use the algorithm in [Appendix C](#) to solve the model. The state vector includes productivity, the capital stock, and active firms, $\mathbf{z}_t = [a_t, Z_{t-1}, K_{t-1}]$. Consistent with the baseline model, we discretize a_t , Z_{t-1} , and K_{t-1} into 10, 15, and 15 evenly-spaced points, respectively. The bounds on the endogenous states, Z_{t-1} and K_{t-1} , are set to $\pm 10\%$ of their respective steady-states.

E SHOCK SIZE IMPLICATIONS

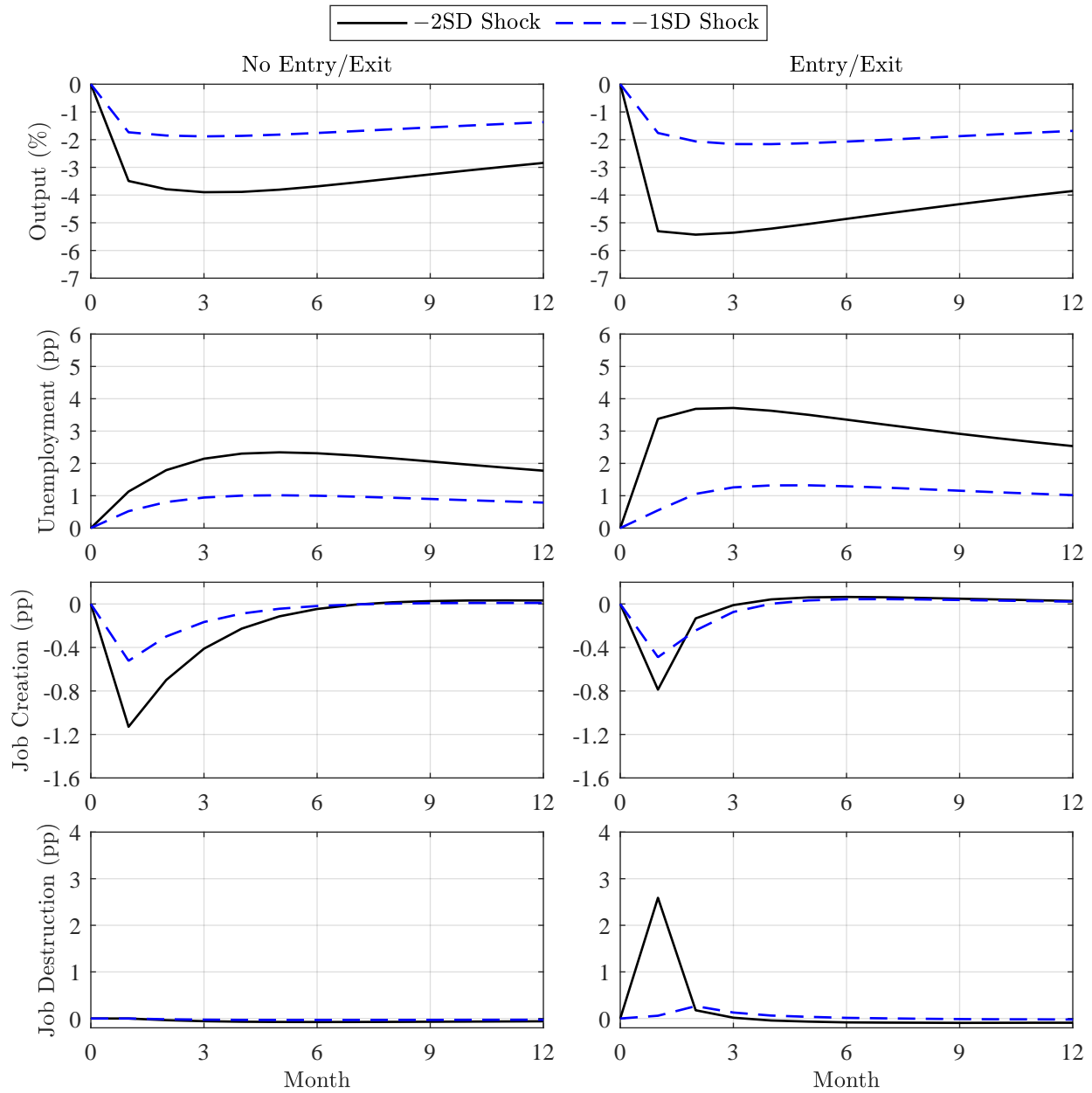


Figure 7: Generalized impulse responses to a technology shock in normal times, $(U_0, Z_0) = (6\%, 1)$.